AN INTRODUCTION TO THE STUDY OF NUMERICAL TRIGONOMETRY

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AN INTRODUCTION TO THE STUDY OF NUMERICAL TRIGONOMETRY

by

E. A. PRICE, B.A.
Assistant Master at the Royal Naval College, Dartmouth
Formerly Assistant Master at Winchester College

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PREFACE

IN reprinting this little work, which formerly appeared under the title of *Examples in Numerical Trigonometry*, some slight alterations and additions have been admitted.

The book is an attempt to introduce the elements of Trigonometry on *heuristic* lines, and is intended for those pupils who do not intend to pursue the study of Mathematics very far. It has for some time been an accepted principle that for such pupils a wider range of subjects treated in a less formal fashion should take the place of that drill in the manipulation of symbols which is essential for the specialist in Mathematics. The author has endeavoured to carry out this principle, and has to a large extent omitted formal proofs and deductions—the theory of the machine which it is proposed to handle.

Even the slight acquaintance with this branch of Mathematics which is aimed at here, will to a large extent miss its true opportunity unless some notion of "*functionality*," or the growth of variable quantities, is acquired, and with this end in view a graphical treatment of the Ratios is introduced, the same idea being applied in dealing with the ratios of the Obtuse Angle.

Among the new features it will be found that headings have been added to the various sections; and the preliminary drawing exercises have been reduced and questions substituted which are intended to call into play the "intuitive"

faculty—the ability to deduce the general rule from few concrete examples by the exercise of the imagination, with, of course, subsequent verification. Additions have also been made to Chapter X.

A chapter on *Logarithms* has been introduced before the general treatment of triangles, but it should be noted that their use is by no means imperative.

Thanks are due to several friends who have kindly assisted in the working out of the answers, and especially to the Director of Naval Education and the Headmaster for permission to make use of questions set in the Examination papers at Osborne.

E. A. P.

January 1918

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CHAPTER I

MEASUREMENT OF ANGLES.

If two straight lines cut one another in such a way that all the angles between them are equal, then each of these angles is called *a right angle.*

A right angle is divided into 90 equal *degrees.* (90°)

A degree is divided into 60 equal *minutes.* (60′)

A minute is divided into 60 equal *seconds.* (60″)

[A second is such a small angle that it is used only in very accurate calculations, *e.g.* in Astronomy. In this book only Degrees and Minutes are used to measure angles.]

1. How many degrees are there in 2 right angles, $\frac{1}{2}$ right angle, $\frac{1}{3}$ right angle, $\frac{1}{10}$ right angle, 0·3 right angle?

2. How many degrees and minutes in $\frac{1}{4}$ right angle, $\frac{1}{12}$ right angle, $\frac{3}{8}$ right angle, $\frac{5}{7}$ right angle (to nearest minute)?

3. Find the sum of each of the following pairs of angles:

(1) 22° 10′	(2) 29° 25′	(3) 38° 51′	(4) 79° 57′
31° 15′	47° 35′	19° 43′	59° 49′

4. If two angles together make up a right angle they are said to be complementary, and each is called the complement of the other.

Calculate the complements of each of the angles in question 3.

5. The three angles of a triangle are together equal to two right angles. If each pair in question 3 represents two angles of a triangle, what is the third angle in each case?

6. If ABC is a right-angled triangle with C a right angle and CD drawn perpendicular to AB, calculate the number of degrees in every acute angle of the figure, and insert the results in a small sketch when A contains (1) 30°, (2) 67°, (3) 43° 21′.

7. Find the number of degrees in each angle of an isosceles triangle if each of the base angles is (1) twice, (2) four times, (3) half the vertical angle. Insert answers in a sketch.

Note. In the two annexed figures the angle ACB is said to be *subtended* at C by the line (or arc) AB, and the angle APB is the angle *subtended* by AB at P.

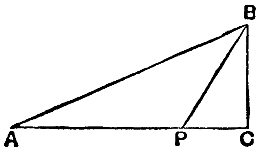


Fig. 1.

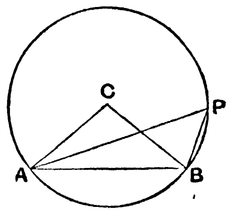


Fig. 2.

8. How many degrees are there in the angle at the centre of a regular 5-sided figure subtended by one of the sides?

How many degrees are there in each of the interior angles, and in each of the exterior angles of the 5-sided figure?

9. Repeat question 8 for (1) an 8-sided, (2) a 9-sided, (3) a 7-sided regular figure.

Note. In recording the *bearing* of one point from another the angle is always measured from a meridian (*i.e.* a North and South line). If a ship lies N. 31° E. of a lighthouse, a man on the lighthouse will find it by looking first due North and then turning through an angle of 31° to the East.

10. If A is N. 31° E. of B, what is the bearing of B from A?

11. If P is N. 39° E. of Q, and R is S. 51° E. of Q, what angle does PR subtend at Q?

12. If X is N. 21° W. of Y, and Z is N. 43° E. of X, and if XY subtends an angle of 30° at Z, what is the bearing of Z from Y?

13. Q is N. 60° E. of R, P is S. 10° E. of Q. If RQ subtends an angle of 35° at P, what is the bearing of P from R?

Note. From a boat B a man is observed at the top of a cliff AC. The angle ABC is said to be the angle of *Elevation* of the man as seen from the boat, and the angle BAD is said to be the angle of *Depression* of the boat as seen from the top of the cliff.

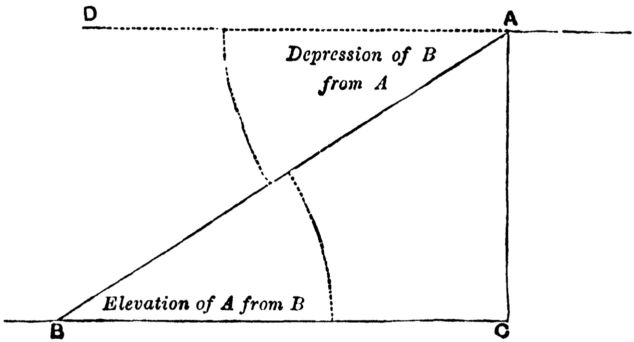


Fig. 3.

Remember that angles of Elevation and Depression are always measured from the Horizontal.

14. There is a church-tower surmounted by a spire. If the angle of elevation of the top of the spire as seen from a certain point on the ground is 53°, and the spire subtends an angle of 29° at the same point, what is the angle of elevation of the top of the tower from the same point?

15. From a boat at sea the angle of elevation of the top of a certain cliff is 37°. What is the angle of depression of the boat as seen from the top of the cliff?

16. From the masthead of a ship the angles of depression of two buoys one behind the other are found to be 19° and 24°. What angle does the line joining them subtend at the masthead?

17. What angle does one-sixth of the circumference of a circle subtend (1) at the centre, (2) at any point on the circumference?

18. What fraction of the circumference is subtended by angles of 120°, 60°, 20°, at points on the circumference?

19. If in Fig. 1 above $C = 90°$, $\angle CPB = 73°$ and $\angle BAC = 25°$, how many degrees are there in each of the angles PBC, ABC, ABP?

20. If in the same figure $C = 90°$, $\angle CPB = x°$ and $\angle BAC = y°$, how many degrees are there in $\angle ABP$?

21. If the edges of the square pyramid in Fig. 8 are each 10 cm. long, how many degrees are there in each of the angles ACB, ACD, BCD, CAK?

Is $\angle AKN$ larger or smaller than $\angle ACN$?

If K were moved along DC towards C, would the angle AKN increase or decrease?

22. From a ship sailing parallel to a straight coastline the angle subtended by the line joining two fixed points on shore is observed. Does this angle vary in size? (Draw a sketch.) If so, when is it greatest?

23. If the angle observed in the last question remained the same for any length of time, would the ship be still moving parallel to the shore? If not, describe carefully her course during that time.

CHAPTER II

THE TANGENT AND ITS USE IN PROBLEMS

Instruments, squared paper, and Table of Natural Tangents required.

Note. Express all results in decimals to three significant figures.

1. Drawing accurately with instruments construct an angle Q of 59°*. Take a point P in four different positions somewhere on one arm of the angle, and draw PN perpendicular to the other arm. Measure PN and NQ in inches and decimals of an inch, and calculate the value of the fraction $\frac{PN}{NQ}$ in decimals for all four positions. Arrange in a table thus:

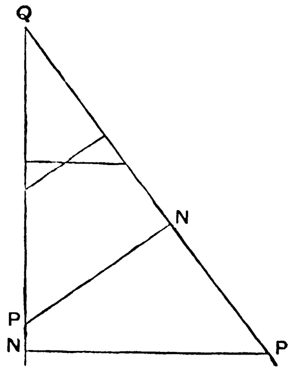


Fig. 4.

		PN	NQ	$\frac{PN}{NQ}$
Q = 59°	1.			
	2.			
	3.			
	4.			

What do you observe with regard to these values of $\frac{PN}{NQ}$?

2. Repeat question 1, making Q equal to 39°.

* The angle Q in the diagram is purposely drawn incorrectly to avoid being copied.

3. If another angle (20° say) were drawn in the same way, would you obtain a similar result?

What do you observe with regard to the value of the fraction $\frac{PN}{NQ}$ for each angle Q? Express *clearly* in words and **remember** the result.

4. The fraction (or ratio) $\frac{PN}{NQ}$ is called the *Tangent* of the angle at Q, and from question 1 is obtained: $\tan 59° = 1{\cdot}66$.

Write down the result of question 2 in the same way.

5. By drawing accurate figures, find the values of $\tan 47°$, $\tan 29{\cdot}5°$, $\tan 34° 24'$ as exactly as possible.

6. Copy the annexed figure accurately, making $QN = 5$ cm., $PNQ = 90°$ and the angles at Q each 10°, the whole angle at Q being 60°.

Measure the length of PN when the angle Q is 10°, 20°, 30°, 40°, 50°, 60°, and calculate the tangent of each of these angles.

7 Turn to the Table of Natural Tangents and search for the results of the last question. Make a comparative table showing the results obtained by drawing and from the printed Tables thus:

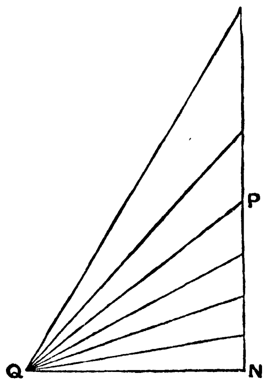


Fig. 5.

Angle	Tangent by Drawing	Tangent from Tables
10°		0·1763
20°		0·3640
30°		etc.
etc.		

8. Make a similar table showing the results of questions 1, 2, and 5 above.

Note that the values given in the Tables are correct to four significant figures only. Tan $20° = 0{\cdot}3639702$ correct to seven figures, but this is not absolutely exact. Four significant figures give results sufficiently accurate for ordinary calculations.

9. Plot on squared paper the values obtained in question 7, *i.e.* draw the graph of $y = \tan x$ for values of x between 0° and 60°. What are the tangents of 0°, 45°, 90°?

Write down the following sentence and insert the word *increases* or *decreases*, whichever is appropriate:

"As the angle x increases from 0° to 90°, $\tan x$...."

10. Draw $\triangle$ABC having C $= 90°$, AC $= 3$ cm., BC $= 4$ cm.

Measure AB in cm. and the angles A and B in degrees and minutes as accurately as possible.

Write down (in decimals to three significant figures) the values of the tangents of the angles A and B as found from the figure.

11. If drawn very accurately, the angle A in the last question should be 53° 8′, and $\tan 53° \, 8' = 1{\cdot}3335$. Search for this in the Table of Natural Tangents and write down the values of $\tan 53° \, 6'$ and $\tan 53° \, 12'$.

12. The angle B in question 10 should have been 36° 52′, and $\tan 36° \, 52' = 0{\cdot}7499$. Search this out and write down the values of $\tan 36° \, 48'$ and $\tan 36° \, 54'$.

13. Write down each of the following and find its value from the Tables: $\tan 56°$, $\tan 56° \, 24'$, $\tan 56° \, 30'$, $\tan 56° \, 27'$, $\tan 28°$, $\tan 28° \, 12'$, $\tan 28° \, 18'$, $\tan 28° \, 15'$, $\tan 16° \, 25'$, $\tan 73° \, 22'$, $\tan 3° \, 51'$.

14. Is the tangent of an angle half the tangent of twice the angle? Compare tan 40° and tan 80° and draw a freehand sketch to illustrate your answer. (See figure of question 6.)

15. From the Tables find the angles whose tangents are 0·4663, 3·0777, 0·3153, 1·4388, 0·4023, 3·6888, 0·1974, 2·3164, 1·1980, 0·6633.

16. If an angle X is such that $\tan X = 0{\cdot}7$ (or $\frac{7}{10}$) it may be constructed by drawing a right angle included between two lines, one of which is seven-tenths of the other. Draw such a triangle having sides 7 cm. and 10 cm., and measure the angle whose tangent is 0·7. Verify the result by reference to the Tables.

17. Find in the same way the angles whose tangents are (1) 0·8, (2) 2·0, (3) 3·5. Verify from the Tables.

Easy Problems

18. From a distance of 100 feet measured along level ground from the foot of a vertical tower the angle of elevation (see p. 3) of the top is found to be 29°. Find by drawing a small freehand sketch and using the Table of Natural Tangents what the height of the tower must be.

19. The angle of elevation of the top of a flagstaff from a point 40 feet from its base is 50°. Calculate its height.

20. If a ladder is placed with its base 5 feet from the bottom of a wall 12 feet high and just reaches the top, what angles does the ladder make with the ground and the wall, and how long must it be?

21. A ship sails 5 miles due West and then 7 miles due North. When it is in this position calculate the distance and bearing of the point from which she started.

22. If the angle of elevation of a monument from a point 200 feet from its base and on the same level is 34°, what is the height of the monument?

23. A vertical pole 12 feet high casts a shadow 20 feet long. What must the angle of elevation of the sun be?

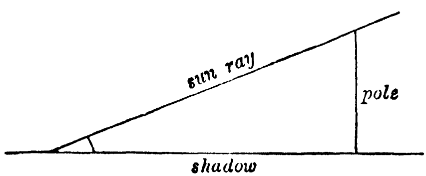


Fig. 6.

Right-angled Triangles

Note. In a triangle ABC the sides opposite the angles A, B, C are usually called a, b, c.

All units of length in the following examples are centimetres. All lengths should be given to four significant figures, all angles in degrees and minutes.

In the triangle ABC having $C = 90°$

24. Find b if $B = 27°$ and $a = 10$ units of length.
25. Find b if $B = 27° \ 12'$ and $a = 10$.
26. Find b if $B = 27° \ 16'$ and $a = 10$.
27. Find b if $B = 27° \ 16'$ and $a = 2{\cdot}5$.
28. Find a if $A = 49° \ 28'$ and $b = 2{\cdot}3$.
29. Find a if $A = 84° \ 24'$ and $b = 12{\cdot}5$.
30. Find A if $a = 4$ and $b = 5$.
31. Find A and B if $a = 8$ and $b = 10$.
32. Find A and B if $a = 2{\cdot}4$ and $b = 1{\cdot}5$
33. Find A and B if $a = 13{\cdot}6$ and $b = 19{\cdot}9$.

Solution of a Triangle

Note. To solve a triangle when three of its sides and angles are given means to calculate the remaining sides and angles.

Solve the triangle ABC if $C = 90°$ and

34. $A = 42° \ 17'$ and $b = 25{\cdot}4$.
35. $B = 29° \ 11'$ and $a = 1{\cdot}62$.
36 $A = 13° \ 53'$ and $a = 10{\cdot}4$.
37. $B = 72° \ 41'$ and $b = 4{\cdot}07$.
38. $a = 12{\cdot}4$ and $b = 9{\cdot}3$.
39. $a = 124{\cdot}5$ and $b = 150$.

Problems

40. From a point 27 feet from the wall of a building a window sill is observed, and its angle of elevation found to be 63° 26′. Find its height above the ground to the nearest inch.

41. To find the range of a distant object C, two men A and B take up positions so that AC is at right angles to AB. If AB is 20 yards, and the angle ABC is found to be 85·4°, find AC.

42. A thin stratum of gold-bearing rock crops out on the level surface and is known to slope down at an angle of 7° 13′. If a shaft is driven vertically down at a distance of $1\frac{1}{2}$ miles from the outcrop, at what depth in feet will it meet the stratum?

43. In order to measure the width of a river a distance AB is marked out straight along the bank; a point C is observed on the bank exactly opposite to A and the angle CBA is measured.

If AB = 50 yards and CBA = 54° 25′, find the width of the river.

44. The rails approaching a tunnel under a certain estuary slope down at an angle of 5° 37′ and are at water level at a distance of 1700 feet from the water's edge. At what depth (in feet) below the surface of the water are they when exactly under the water's edge?

45. In a triangle ABC having C = 90°, AC = 5 inches and BC = 12 inches, BD is drawn at right angles to AB to meet AC produced in D. Find the length of CD.

46. A chord 6 cm. long is 2 cm. from the centre of a circle. What angle does it subtend (see p. 2) at the centre?

47. If a chord 10 cm. long subtends 140° at the centre of a circle, how far must it be from the centre?

48. From a point on level ground 75 feet from the base of a flagstaff, the angles of elevation of the top and of the point at which a yardarm is fixed are found to be 49° 37′ and 38° 29′.

Find the height of the top and of the yardarm above ground.

49. On a map a piece of straight road between the 100-foot and 400-foot contours seems to be $\frac{1}{4}$ mile long. What is its slope and how many yards long is it really?

50. The poles for a certain Wireless Telegraphy Station consist of three parts fixed vertically one above the other, each part

being 50 feet long. The whole is held erect by wire stays fixed to the top of each part and to points in the ground 75 feet away from the base. Find the angles which each of the wire stays makes with the ground and with the poles. Find also the angles between the three wire stays fixed to the same point on the ground. Draw a sketch and insert the answers.

51. From a ship sailing up a river the angle of elevation of a point on a bridge 176 feet above the water and straight in front is found to be 29° 14′.

If the ship is moving at a rate of 5 miles an hour, in how many seconds will it be exactly under the bridge?

52. The figure is a sketch for the following problem:

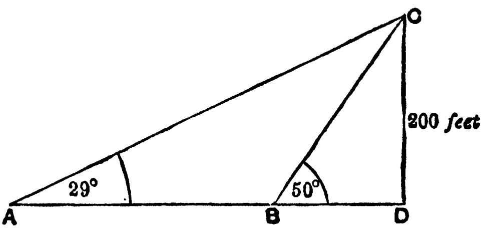


Fig. 7.

From a boat the angle of elevation of an object on the top of a cliff 200 feet high is found to be 29°. The boat sails directly towards the object, and after a certain interval the angle of elevation is found to be 50°. How many feet did the boat sail?

[Calculate ∠ ACD, ∠ BCD, AD, BD; then AB can be found.]

53. Repeat question 52, making ∠ A = 27° 13′, CD = 274 feet and ∠ CBD = 63° 29′.

54. The angle of elevation of the top of a tower is observed to be 25°, and from a point 100 feet nearer to the foot of the tower it is 43°. Find the height of the tower.

55. Repeat question 52, with angles 18° 47′ and 51° 29′ and distance 124 feet.

56. The figure represents a square pyramid each of whose edges is 10 cm. long. It is required to find the angle (ABN)

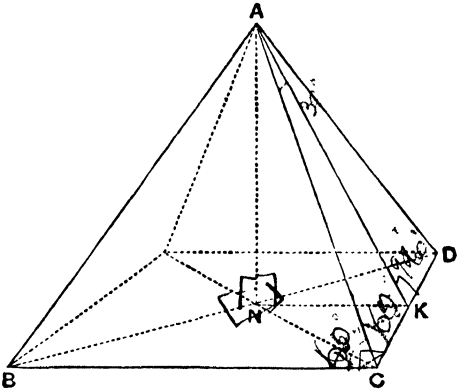


Fig. 8.

between one slant-edge and the base, and also the angle (AKN) between one face and the base.

Note that ANB, ANC, ANK, AND are right angles. Calculate BN and AN by Pythagoras' Theorem.

57. Calculate the height of a regular tetrahedron (triangular pyramid) whose edges are each 10 cm. long. Find also the angles between slant-edge and base, and between two faces.

Note NK = $\frac{1}{3}$BK.

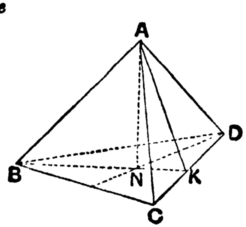


Fig. 9.

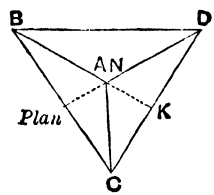


Fig. 10.

CHAPTER III

THE SINE AND ITS USE IN PROBLEMS

Instruments, squared paper, and Table of Natural Sines required.

1. Construct an angle Q of 59° carefully with instruments. Take a point P in four different positions somewhere on one arm of the angle and draw PN perpendicular to the other arm. Measure PN and PQ for each position of P and calculate the value of the fraction $\frac{PN}{PQ}$ in each case. Tabulate the values of PN, PQ, and $\frac{PN}{PQ}$. [Drawings for Chap. II may be used.]

2. Repeat question 1, making $Q = 39°$.

3. What do you observe with regard to the fraction $\frac{PN}{PQ}$ in each case? Express *clearly* in words and **remember** the result.

4. The fraction (or ratio) $\frac{PN}{PQ}$ is called the *Sine* of the angle Q. What is the value of sin 59° ?

Write down the result of question 2 in the same way.

5. By drawing accurate figures, find the values of sin 47°, sin 29·5°, sin 34° 24′ as exactly as possible.

6. Copy the annexed figure accurately, making the radius of the circle 10 cm. long and putting in radii at intervals of 10°.

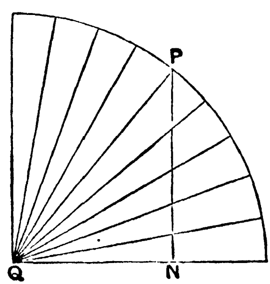


Fig. 11.

If P be the end of a radius and PN be drawn perpendicular to NQ, then by measuring PN in all positions the sines of the angles 10°, 20°, 30°, 40°, 50°, 60°, 70°, 80° can be calculated.

Find these values and make a table. What is the value of sin 0° and of sin 90°?

7. Plot these values on squared paper (*i.e.* draw the graph of $y = \sin x$ for values of x between 0° and 90°). From the graph find the values of sin 47° and sin 29·5°, and compare with the results of question 5. Does $\sin x$ increase or decrease as x increases from 0° to 90°? Answer quite clearly.

8. Compare the results obtained in question 6 with the values given in the Table of Natural Sines. Arrange in a table thus:

Angle	Sine	
	by Drawing	from Tables
0°		0·0000
10°		0·1736
20°		0·3420
etc.		etc.

9. In Ex. 10, Chap. II a triangle ABC having $C = 90°$, $AC = 3$ cm. and $BC = 4$ cm. was drawn, and it was found that $AB = 5$ cm. and $A = 53°$ about. Draw a neat freehand sketch of this triangle. If drawn very accurately the angle A should be 53° 8′, and sin 53° 8′ = 0·8000 (or $\frac{4}{5}$). Search for this in the Table of Natural Sines and write down the value of sin 53° 6′, and of sin 53° 12′.

10. The angle B in the triangle above should be 36° 52′, and sin 36° 52′ = 0·5999 (or $\frac{3}{5}$ nearly).

Search for this in the Table of Natural Sines, and write down the value of sin 36° 48′ and of sin 36° 54′.

11. Write down each of the following and find its value from the Tables:

sin 35° 12′, sin 35° 18′, sin 35° 15′, sin 35° 13′,
sin 72° 55′, sin 43° 9′, sin 10° 53′, sin 81° 49′.

12. Is the sine of an angle half the sine of twice the angle? Compare sin 40° and sin 80° and draw a freehand sketch to illustrate your answer. (See figure of question 6.)

13. From the Tables find the angles whose sines are

0·4226, 0·9455, 0·8141, 0·2890,
0·2898, 0·9795, 0·9701, 0·7202.

14. By drawing a right-angled triangle with one side 5 cm. long and the hypotenuse 10 cm. long and measuring the angles, find the angle whose sine is ·5 $(=\frac{5}{10})$. Verify from the Tables.

15. Find in the same way the angles whose sines are (1) 0·4, (2) 0·75, (3) 0·43. Verify from the Tables.

Right-angled Triangles

16. If in $\triangle$ ABC having C = 90° (draw freehand sketch)

(1) B = 27°, $c = 10$ units, find b.
(2) B = 27° 12′, $c = 10$, find b.
(3) B = 27° 16′, $c = 10$, find b.
(4) B = 27° 16′, $c = 2·5$, find b.
(5) A = 43° 37′, $c = 12·2$, find a.
(6) A = 11° 22′, $c = 11·22$, find a.

17. In the triangle ABC, C = 90°, B = 63° 28′ and $c = 75$ cm. Find b, a (by Pythagoras) and A. Verify by drawing to scale.

18. Solve the triangle ABC if C = 90° and

(1) A = 72° 14′, $c = 21$. (Do not use Pythag.)
(2) B = 25° 23′, $c = 10·4$.
(3) B = 42° 13′, $c = 105·2$.
(4) A = 11° 19′, $c = 19·32$.

Problems

19. A straight road rises 48 feet in 400 yards measured along the road. Find its inclination to the horizontal.

20. What is the angular slope of a railway line whose gradient is 1 in 100? (The 100 is measured along the rails.)

21. What angular slopes are represented by the boards at the side of a railway line which are shown in the figure?

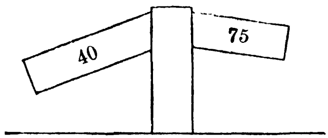


Fig. 12.

22. A ladder 25 feet long is placed against a wall and makes an angle of $73^\circ\ 22'$ with the ground. To what height does it reach? Answer to nearest inch.

23. What is the height of an isosceles triangle whose equal sides are each $2{\cdot}5$ inches long and whose base angles are $17^\circ\ 35'$ each?

24. A lighthouse 13 miles away is observed from a ship to be 19° off the course of the ship. If the ship continues on her course, how far from the lighthouse will she be when she is nearest to it?

25. A rod 15 inches long hangs from a nail in the wall. If it is pulled away so that it makes an angle of $25^\circ\ 33'$ with the wall, what is the distance of the lower end from the wall?

26. What angle will the rod in the last question make with the vertical when its lower end is 8 inches from the wall?

27. A trap-door is 3 feet 3 ins. wide from hinge to opposite edge. If it is raised through four-fifths of a right angle, how high above the ground will the outer edge be?

28. A framework of four rods each 1 foot long, hinged at the ends, and with elastic diagonals, is laid upon the table. Find the lengths of the diagonals when the angle between two adjacent sides is (1) $45^\circ\ 22'$, (2) $145^\circ\ 22'$.

CHAPTER IV

THE COSINE AND ITS USE IN PROBLEMS

Definition. If from any point P in one arm of an angle Q a perpendicular PN be drawn to the other arm, the ratio $\frac{QN}{PQ}$ is called the *cosine* of the angle Q.

Remember this definition.

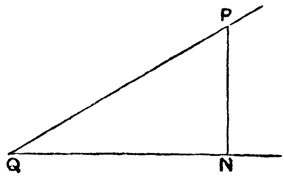


Fig. 13.

1. Draw any acute angle accurately with instruments and show by careful measurements that the cosine of this angle remains the same for any four different positions of the point P. Arrange results in a table.

2. Find by accurate drawing the value of cos 59° and of cos 39° (cos 59° means the cosine of an angle of 59°).

3. Find by accurate drawing the angles whose cosines are (1) 0·5, (2) 0·75, (3) 0·36.

4. Copy the annexed figure accurately making the radius 10 cm. long and putting in radii at intervals of 10°.

By drawing PN in all positions and measuring NQ, find the cosines of the angles 0°, 10°, 20°, 30°, ... 90°. Tabulate the resulting values.

Does the cosine increase or decrease as the angle increases from 0° to 90°?

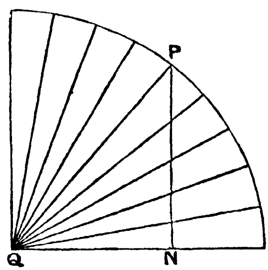


Fig. 14.

5. Compare the results obtained in question 4 with the values given in the Table of Natural Cosines.

To the table of results in that question add a column showing the values given in the printed Table.

6. From the Table of Natural Cosines find and write down the values of cos 56°, cos 56° 24′, cos 56° 30′.

Now cos 56° 27′ lies between the last two values. Find it, and write it down.

7. Find from the Tables the values of

cos 28° 15′,	cos 16° 25′,	cos 73° 21′,
cos 81° 51′,	cos 53° 8′,	cos 12° 28′.

8. In a triangle ABC having $C = 90°$ and

(1) $A = 27°$, $c = 10$, find b.

(2) $A = 43° \, 22'$, $c = 10$, find b.

(3) $B = 19° \, 25'$, $c = 8{\cdot}8$, find a.

(4) $B = 85° \, 17'$, $c = 12{\cdot}2$, find a.

PROBLEMS

9. A wire 100 feet long is stretched from the top of a flagstaff to a point in the ground. If it makes an angle of 63° 29′ with the ground, how far is this point from the foot of the flagstaff?

10. A ship has sailed North-West for 21 miles. How far North of her starting-point is she?

11. A straight road running uphill at an angle of 9° 35′ with the horizontal is 1 mile long. How many yards long will it appear to be on the map?

12. A ladder 25 feet long leans against a wall making an angle of 59° 43′ with the ground. How far is its foot from the foot of the wall?

13. What is the length of the base of an isosceles triangle whose equal sides are 2·5 inches long and whose base-angles are each 17° 34′?

14. Gloucester is 79 miles N. 29° W. of Southampton. How far N. of Southampton is Gloucester?

15. A rod 3 feet long hangs from the ceiling. How far is the lower end from the ceiling when the rod makes an angle of 69° 26′ with the vertical?

16. What is the length of the shortest side of a set-square with angles of 90°, 60°, 30°, if the longest side is 15 cm. long?

17. A flagstaff whose top when vertical is 70 feet above the ground is found to lean to one side making an angle of 82° 20′ with the ground.

What is the length of its shadow when the sun is vertically overhead?

18. A roof is made of pieces of corrugated iron leaning against a wall and supported at their lower ends by poles. If the length of each piece of iron is 6 feet 6 inches, and it makes an angle of 32° with the horizontal, what width will be kept dry when rain falls vertically?

19. If the latitude of New York be taken as 40° N. and the radius of the Earth as 4000 miles, calculate the distance of New York from the axis NS of the Earth. Through what distance does New York revolve each day? (See Fig. 15.)

20. How many miles a minute is London moving through space owing to the spin of the Earth on its axis? Take London's latitude as 51° 30′.

21. What is the distance round the world in latitude 60°? On Mercator's Projection of the World the width of the Atlantic Ocean in latitude 60° N. appears to be the same as its width on the Equator.

What is the true relation between these distances?

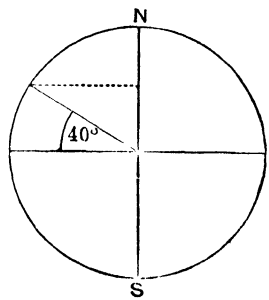


Fig. 15.

CHAPTER V

THE SECANT, COSECANT, AND COTANGENT

Note. These ratios are shortened to *sec* A, *cosec* A, *cot* A.

1. Find the value of sec 36° from the Table of Natural Secants. Find also the value of cos 36°. Multiply these together and give the answer correct to four significant figures.

2. Repeat twice with any other angles.

What do you observe with regard to the secant and cosine of any one of these angles?

Note. $\frac{2}{3} \times \frac{3}{2} = 1$, and $\frac{2}{3}$ is called the *reciprocal* of $\frac{3}{2}$.

3. Find in a similar way some relation between the cosecant and some other ratio known already.

4. Repeat for the cotangent and tabulate the relations found in the first four questions. The results should be remembered.

5. If ABC is a triangle with C = 90°, express cosec A, sec A and cos A each as a ratio of two sides.

6. Find from the Tables and write down the values of sec A, cosec A and cot A when A is
56°, 56° 24′, 56° 30′, 56° 27′;
28°, 28° 12′, 28° 18′, 28° 14′;
16° 25′, 73° 21′.

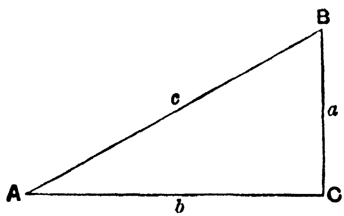


Fig. 16.

7. Write down the following and complete it by inserting the words "increases" and "decreases":

"As the angle A increases from 0° to 90°, sec A..., cosec A..., cot A...." Illustrate with a freehand drawing.

PROBLEMS

Note. Use *secants* in questions 8 to 11.

8. A wire is stretched from the top of a vertical pole to a point in the ground 20 feet from the base. How long must the wire be if it makes with the ground an angle of (1) 28°, (2) 53° 19′, (3) 61° 58′?

9. The straight line joining the tops of two vertical poles 31 feet apart is found to make an angle of 19° 23′ with the horizontal. What is the distance between the tops?

10. From the top of a cliff 470 feet high the angle of depression of a boat is found to be 23° 14′. How far is the boat from the top of the cliff? [Find the complement.]

11. Find the radius of the circle circumscribing a regular five-sided figure whose sides are each 12 cm. long. [Calculate the angle at each corner.]

Note. Use *cotangents* in questions 12 to 16.

12. A ladder makes an angle of 41° 26′ with a vertical wall. If its lower end is 10 feet from the base of the wall, to what height does the top reach?

13. One side of a rectangle is 15 inches long and the diagonal makes an angle of 17° 23′ with the other side. Find the length of the other side.

14. How far from a tower 110 feet high must a man be standing if the angle of elevation of the top is 35° 4′?

15. A is 25 miles due N. of B, and C is due E. of B. If AB makes an angle of 13° 58′ with AC, find the angle C and BC.

16. P is a point on the circumference of a circle whose diameter is XY. If the straight line from P to X is 53 inches long and subtends an angle of $47° 13'$ at Y, find the length of the chord YP.

Note. Use *cosecants* in questions 17 to 20.

17. How long must a ladder be which just reaches a window 15 feet above the ground, if it makes an angle of $53° 19'$ with the ground?

18. What is the diameter of a circle in which a chord 5 inches long subtends an angle of $25° 28'$ at the circumference? [Draw the diameter through one end of the chord.]

19. If the slope of a road is $4° 6'$, what is its gradient? (Cf. Ex. 20, Chap. III.)

20. If the line joining the tops of two vertical posts makes an angle of $17° 13'$ with the horizontal, and if one post is 6 feet longer than the other, what is the distance between the tops?

Exercises on the Six Ratios

21. Draw a freehand sketch of a triangle ABC having $C = 90°$, $a = 4$ inches, $b = 3$ inches. From this figure find the six trigonometrical ratios of A and of B.

Find the size of the angle A by looking up sin A in the Table of Natural Sines, and verify the remaining five ratios by searching for this angle in the appropriate Tables. Repeat for the angle B and arrange all results in columns.

22. Repeat question 21 for a triangle having $C = 90°$, $c = 40$ cm. and $b = 9$ cm.

23. Observing that the angles A and B in questions 21 and 22 are complementary, what do you note with regard to the ratios of complementary angles?

24. If $\tan A = \frac{5}{12}$, find (by drawing a freehand sketch) the other five ratios of A and write them down. Verify from the Tables as before.

25. If $\sin a = \frac{7}{25}$, find the other ratios and verify.

26. Find the sine, cosine and tangent of 35° from the Tables. Calculate the value of $\dfrac{\sin 35^\circ}{\cos 35^\circ}$, and of $\sin^2 35^\circ + \cos^2 35^\circ$. [$\sin^2 35^\circ$ means the square of the sine of 35°.]

What do you observe with regard to these values?

27. Repeat question 26 with any angle chosen at random. Do the same relations hold good?

28. *Prove* that these relations are true for any acute angle by drawing a sketch of a right-angled triangle and making use of Pythagoras' Theorem.

29. Making use of your tables if necessary, draw a rapid freehand sketch of the graph of $\sin A$ when A has values between 0° and 90°.

30. As A increases from 80° and becomes more and more nearly equal to 90°, to what value does PN (see figure of Ex. 6, Chap. III.) more and more nearly approach?

What then is the ultimate value of $\sin 90^\circ$?

Write down the values of $\cos 90^\circ$, $\tan 90^\circ$, $\sin 0^\circ$, $\cos 0^\circ$, $\tan 0^\circ$.

31. Draw rapid freehand sketches of the graphs of $\cos A$ and $\tan A$ for values of A between 0° and 90°.

32. What can be asserted with regard to the greatest and least values of the *sine* of an acute angle?

What can be asserted with regard to the *cosine* and *tangent* of an acute angle?

33. By means of the Tables find a relation between $\tan^2 35^\circ$ and $\sec^2 35^\circ$.

34. Choose an angle at random and show that the relation is still true.

35. Prove as before (question 28) that this relation is true for any acute angle.

36. Find and prove a relation between $\operatorname{cosec}^2 A$ and $\cot^2 A$ when A is an acute angle.

CHAPTER VI

MISCELLANEOUS PROBLEMS INVOLVING RIGHT-ANGLED TRIANGLES

Note. **At this stage it is most essential that the pupil should be able to recognize immediately the ratios of the angles in a right-angled triangle and readily to express the length of one line in terms of another. Constant practice with figures similar to those in questions 1 and 6 below is recommended for pupils who hesitate.**

Revise Ex. 6, *Chap. I.*

1. In the figure $C = 90°$ and p is perpendicular to c cutting it into two parts x and y.

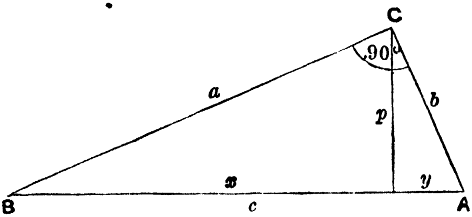


Fig. 17.

Which other angle in the figure is equal to B, and why? Show that the whole triangle and the two triangles into which p divides it are all equiangular (and therefore the same shape). Now $\frac{p}{a} = \sin B$. Write down two other fractions equal to sin B.

2. Write down three different expressions for cos B, tan B, sec B, cosec B, cot B, in the figure of question 1.

3. Write down three expressions for each trigonometrical ratio of the angle A in the figure of question 1.

4. If in the figure of question 1, $a = 20$ cm. and $p = 12$ cm., find B, A, b, x, y and c, and verify the results of questions 1—3.

5. If in the figure of question 1, $b = 6{\cdot}5$ cm. and $p = 6$ cm., find A, B, a, x, y and c.

6. In the figure $\frac{\text{AN}}{x} = \cos a$,

$$\therefore \text{AN} = x \cos a.$$

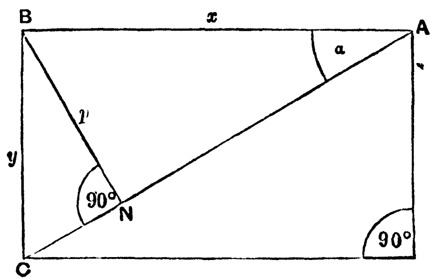


Fig. 18.

Write down in a similar way the values of p, y, AC, CN, in terms of x and a.

7. In the figure of question 1 what lengths are represented by $a \cos$ B, $b \sin$ A, $c \cos$ B, $a \cos$ A, $p \tan$ B, $p \cot$ B, $p \sec$ A, p cosec A, $b \sin$ B?

8. In the figure of question 6 what lengths are represented by $x \sin a$, $x \tan a$, $p \cot a$, $y \cos a$, $p \tan a$?

Without using Pythagoras' Theorem solve the triangles in which $C = 90°$ and

9. $B = 24° 19'$, $a = 10{\cdot}4$.
10. $A = 29° 53'$, $a = 20{\cdot}1$.
11. $A = 44° 31'$, $c = 2{\cdot}4$.
12. $a = 24{\cdot}7$, $b = 14{\cdot}6$.
13. $c = 140$, $a = 77$.
14. $A = 72° 13'$, $b = 15{\cdot}5$.
15. $B = 34° 37'$, $c = 1200$.
16. $b = 5{\cdot}7$, $c = 11{\cdot}4$.
17. $c = 16{\cdot}75$, $a = 3{\cdot}35$.
18. $c = 3a$.

Note. In the following questions the answers may be obtained in a variety of ways, but by using the appropriate ratio the answer may be arrived at by one operation.

19. From the top of a cliff 250 feet high the angle of depression of a boat is found to be 17° 43'. How far is the boat from the top of the cliff?

20. A ship sails 17 miles due N. and then 25 miles due E. In what direction does the starting-point lie?

21. The angle of elevation of the top of a tree is found to be 29° 11'. If the observer is 120 feet from its base, how high is the tree?

22. The string of a kite (supposed to be straight) is 225 feet long and is inclined to the ground at an angle of 52° 5'. Find the height of the kite above the ground.

23. What is the slope of a railway line whose gradient is 1 in 60?

24. Find the length of one of the equal sides of an isosceles triangle whose base is 12·4 cm. long and whose base-angles are each 41° 28'.

25. Find the radius of a circle in which a chord 3·4 cm. long subtends an angle of 104° at the centre.

26. From the top of a flagstaff 108 feet high a wire 125 feet long is stretched to the ground. What is the angle between the wire and the flagstaff?

27. In a triangle PQR, $PQ = 12 \cdot 2$ cm. and $Q = 39° \cdot 37'$. What is the length of the straight line PN drawn from P perpendicular to QR?

28. Find also the length of QN in the last question.

29. If in the question above, $QR = 17 \cdot 8$ cm., what is the length of NR? Hence find the angle at R.

30. An aeroplane is observed from a point A and its angle of elevation is found to be 67° 32′. If at the same instant it is exactly above B (on the same level as A), and $AB = 75$ yards, how high is it above B?

31. The angle A of a parallelogram ABCD is 31° 3′ and the perpendicular distance between AD and BC is 3·52 inches. Find the length of the side AB.

32. PN is a tangent to a circle whose centre is O, N being on the circumference. If the radius is 5·7 cm., and $OP = 9 \cdot 5$ cm., what angle does PN subtend at the centre?

33. If a door 4 ft. 2 inches wide is opened through an angle of 112° 42′, how far (in a straight line) is the bottom corner from its position when the door was closed?

34. What is the radius of a circle which touches the five sides of a regular pentagon if each side is 5 cm. long? [Find the angle at the centre.]

35. Find the radius of the circle which passes through the five angular points of a regular pentagon whose sides are each 5 cm. long.

36. Repeat questions 34 and 35 for a regular hexagon (6 sides).

37. Repeat for a regular decagon (10 sides).

38. A ship is sailing N. 11° 15′ E. at 9 miles an hour. At what rate is she moving due N.?

39. From a point 27 feet from the base of a pole and on the same level, the angle of elevation of the top is 63° 26′. Calculate its height to the nearest inch. Find also the distance of the point of observation from the top.

40. Find the angle of elevation of the top of the pole in the last question from a point 20 feet from the base.

41. A rod 5 inches long is hinged at a point on the ground, and from the other end hangs a plumb-line. What angle does the rod make with the plumb-line when the top of the rod is (*a*) 4 inches above the ground, (*b*) 2 inches above the ground?

42. A pole is 126 feet high. A man 6 feet high stands at a distance of 100 feet from the foot of the pole and observes the angle of elevation of the top of the pole. What does he find the angle of elevation to be?

43. A diameter AB of a circle is 5 cm. long. P is a point on the circumference 2·34 cm. from A. Find the angle subtended by PA at B.

44. The shadow of a vertical stick 4 feet high is found to be 5·72 feet long. Find the altitude of the sun.

45. A lies 5 miles due North of B, and C lies $6\frac{1}{2}$ miles due East of B. Calculate the distance and bearing of C from A.

46. At a certain Marconi Station a vertical pole 96 feet high is made in three equal sections and supported by wire stays stretched from the top of each section and fixed to points in the ground. If wires from the top of each section are fixed to a point on the ground 25 feet from the base of the pole, calculate the angles which they make with the ground.

47. Find the height of an isosceles triangle whose base is 10 cm. long and whose vertical angle is 35° 28′.

48. In the figure, CP represents the connecting-rod and OP the crank of a steam engine. P revolves round O, and C moves backwards and forwards along OC.

If CP is 40 inches long and OP is 12 inches long, find the angle at C when OP is at right angles (1) to CP and (2) to OC.

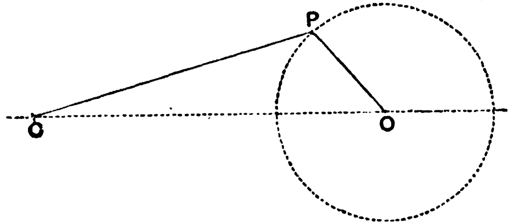


Fig. 19.

49. ABC is a triangle right-angled at C. If BC = 17·3 feet and if ABC = 30° 27′, calculate the length of AB to the nearest inch.

50. From the top of a cliff 300 feet high the angle of depression of a ship is 13° 46′ and the angle of depression of a rock between the ship and the shore is 32° 10′. Calculate the distance of the ship from the rock (to the nearest foot).

51. On one occasion while the Atlantic cable was under repair, it was found that when the grappling iron seized the cable on the ocean bed, 4890 fathoms of hawser had been paid out, and that when drawn taut the hawser made an angle of 72° with the horizontal. What was the depth of the water at this spot in fathoms?

52. How many degrees are there in an angle in a semicircle? In a semicircle of diameter 10 inches, a chord 4 inches long is put, one end of which coincides with one end of the diameter. Calculate (*a*) the angle between the chord and the diameter, and (*b*) the length of the chord joining the other ends of the diameter and chord.

53. A diameter AB of a circle bisects a chord CD at right angles at O. The angle $OCA = 50°$. Find the lengths of AO and BO, given that the chord CD is 20 inches long. What are lengths of AC and BC?

54. A lighthouse bears 5 miles due N. of a cruiser steaming N. 18·6° E. By how many miles does the cruiser clear the lighthouse?

55. A cruiser and a torpedo boat set out at the same time from point C. The cruiser steams at the rate of 20 knots in a direction N. 15° 24′ E., and the torpedo boat at the rate of 30 knots in a direction N. 74° 36′ W. Calculate the distance and bearing of the torpedo boat from the cruiser at the end of 30 minutes.

56. A, B, and C are buoys. The bearing of B from A is N. 47° E., and the bearing of C from A is S. 43° E. B is known to be 5 miles from A, whilst C is due S. of B. Find the distances of C from B and A.

57. A flagstaff on the far bank of a river is seen by a man immediately opposite to subtend an angle of 57°, whilst on retiring 100 ft. the elevation is only 35°. Find the breadth of the river to the nearest foot.

58. The Rock of Gibraltar is 1396′ in height; the angle of elevation of its summit from a ship is 22°: how far will the ship have to move directly towards it before its angle of elevation is 31°?

59. A straight railroad rises 2·4 feet in every 100 yards of rail: find its inclination to the horizontal.

60. A straight line from Southampton pier to Ryde pier subtends 90° at Portsmouth pier; Southampton pier lies 15·2 miles N. 46° W. from Ryde pier, and Portsmouth pier bears N. 28° E. from Ryde pier; find the distance between Portsmouth and Ryde piers.

61. At Southampton on Midsummer's Day the altitude of the sun is 62° 45′. If a mast is 100 feet high, find the length of its shadow.

Find also the length of its shadow on December 21st, when the altitude of the sun is 15° 45′.

62. From the top of a vertical cliff 200′ high the angle of depression of a boat anchored out in the bay is 46° 28′. Find how far the boat is from the foot of the cliff.

63. A kite is held by a string 100 yards long, and its angle of elevation is found to be 50° 16′. What is the vertical distance of the kite from the ground? (Neglect the sag of the string.)

64. A ship X is 10 miles S. 52° W. of a harbour at the moment that another ship Y is leaving the harbour. If Y steams S. 38° E. at 8 knots and the ships meet in 2 hours, find X's course.

65. From the top of a tower A, 60 feet high, the angle of depression of the foot of another tower B is 23° 16′; whilst from the foot of A the elevation of the top of B is 45° 17′. Find how far A is from B, and also the height of B to the nearest foot.

66. A ship on leaving port steams 10 miles N. 50° 11′ E., and then 5 miles N. 39° 49′ W. Find the distance she must go, and the course she must steer, to reach port again.

67. In a circle of radius 2·7 inches find the length of a chord which subtends an angle of 163° 10′ at the centre. Find also how far this chord is distant from the centre of the circle.

68. From a ship sailing on a course due N. a lighthouse, 13 miles away, is observed to bear N. 19° 34′ E. When the ship is nearest to the lighthouse, how far from it will she be?

What will be the bearing of the lighthouse from the ship when they are 7 miles apart?

69. (The diagonals of a rhombus bisect each other at right angles and also bisect the angles through which they pass.) Make use of these properties to find the angle at B of the rhombus ABCD, the length of each side of which is 10 inches, the length of the diagonal AC being 5·234 inches.

70. Observations were taken from a point 500 feet from the foot of a tower with a flagstaff on the top, and the angles of elevation of the top of the tower and of the top of the flagstaff were found to be 28° 15′ and 30° 47′ respectively. Find (1) the height of the tower and flagstaff; (2) the height of the tower alone; and hence (3) the height of the flagstaff alone.

71. A ship sails 4 miles due West and then 3 miles due North; find its distance from the starting point and its bearing from that point.

72. A bears 10 miles due N. of C, and B bears N. 25° E. of C, and B bears S. 65° E. of A. Find the distances of B from A and C.

73. A lighthouse 10 miles away is seen to be 15° off a ship's course. At what distance will the ship pass the lighthouse if she holds her course?

74. At 10 a.m. a ship is observed from a lighthouse to bear 9 miles N. 57° 32′ E. and is known to be sailing S. 32° 28′ E. At 11 a.m. her bearing is S. 60° 45′ E. Find (1) rate of ship's sailing; (2) distance of lighthouse at second observation.

75. In a triangle ABC, the angle B = 36° 52′, AB = 4·5 inches, and BC = 4·68 inches.

By drawing AD perpendicular to BC, and calculating the length of AD and BD, find the length of DC and AC, and the size of the angle C.

Make a table of the sides and angles of the triangle ABC.

76. A theodolite was used to determine the height of a flagstaff and the following data were obtained :—

Height of theodolite telescope above ground 3 ft. 4 in.
Distance of theodolite from foot of flagstaff 120 ft.
Angle of elevation of the top of the flagstaff 27·5°.

Calculate the height of the flagstaff.

77. A and B are two points on opposite sides of a tower. The distance AB is 140 feet. The angles of elevation of the top of the tower from A and B are respectively 62° and 31°. Calculate the height of the tower.

78. O and P are points on a straight stretch of shore 1 mile apart, and O bears N. 74° W. of P. From a ship at sea O bears N. 15° W. and from the same ship P bears N. 75° E. Calculate the distance of the ship from O and also its distance from the nearest point of the shore.

79. The ropes of a swing are 25 feet long and the height of the seat above the ground at the highest and lowest points are 14 feet and 3 feet respectively.

What is the angle through which the swing moves from side to side?

80. A flagstaff 90 feet high subtends an angle of 45° at a point A on the ground due South of it. Find the angle of elevation of the top of the flagstaff at a point B, 120 feet due East of A.

81. The Tay Bridge extends North and South a distance of 1·764 miles. A man walks due East from the North end of the bridge until the bridge subtends an angle of 33° 17′ at his eye. How far is he now in a straight line from the South end of the bridge?

82. A flagstaff consists of two poles one fixed above the other. From a point 64 feet from the base the angles of elevation of the tops of these poles are found to be 40° 29′ and 61° 51′. Find the length of each pole.

83. From the top of a cliff 320 feet above sea level the angles of depression of two boats in a line are found to be 11° 21′ and 34° 19′. Find their distance apart.

84. From a ship sailing due S. at 12 miles an hour the directions of two objects on shore are observed to be S. 29° 14′ W. and S. 43° 7′ W. After sailing for 25 minutes they are observed to be in a line due W. of the ship. What is the distance between them and how far was the ship from each of them at the first observation?

85. (See figure of Ex. 56, Chap. II.) If A is the vertex of a square pyramid whose base is BCD, and if BC = 10 cm. and AB = 12 cm., find (1) the angle between AB and BC, (2) the angle between AB and BD, (3) the inclination AKN of a side-face.

86. Find the height of a square pyramid each of whose edges is 5 inches long. Find also the inclination of (1) a side-edge, (2) a side-face to the base.

87. If the edges of a rectangular block are 2, 3 and 6 inches respectively, find the angle between a diagonal of the block and the diagonal of each face.

88. The angle of elevation of a tower 250 feet high is observed from A to be 20°. How far is A from its foot? If A is due E. of the tower and due N. of a point B from which the tower bears N. 40° W., how far is B from the foot? Find the elevation of the top from B.

89. Repeat question 88 making height of tower x feet and 18° the angle of elevation from A.

90. P is 1000 yards due N. of Q and on the same level. An airship is observed from P and Q simultaneously. From P its elevation is 23° and it bears due W. From Q it bears N. 42° W. Find its height above the ground.

CHAPTER VII

AREAS

1. What is the area of a rectangle whose adjacent sides are (1) 5 cm. and 10 cm., (2) 131·4 yards and 101·2 yards?

2. Draw a sketch of each of the rectangles above and draw a diagonal.

By considering these figures find the area of a right-angled triangle whose shorter sides are (1) 5 cm. and 10 cm., (2) 131·4 yards and 101·2 yards.

3. What is the area of each of the right-angled triangles ANB, ANC in the figure? What is then the area of the whole triangle ABC?

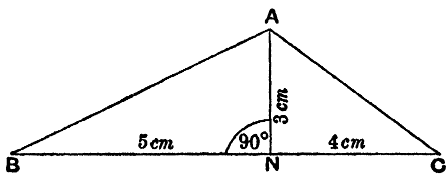


Fig. 20.

4. How is the area of a triangle found when the base BC and the height AN are known? Express the answer *quite clearly* in words.

5. If two sides of a parallelogram are 7 cm. and 5 cm. long respectively and one of its angles is 40°, what are the perpendicular distances between the opposite sides? The parallelogram is twice the triangle ABC. Calculate its area. (See Fig. 21.)

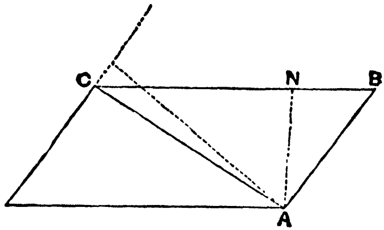


Fig. 21.

6. How is the area of a parallelogram found when the sides and the perpendicular distances between them are known?

7. How is the area of a parallelogram found when two sides (x, y) and the included angle (A) are known?

8. How is the area of a triangle found when two sides and the included angle are given?

9. Find the area of each of the following triangles:

(1) $a = 5$ cm., $b = 4$ cm., C $= 30°$.

(2) $b = 10$ cm., $c = 3$ cm., A $= 36° \ 52'$.

(3) $b = 6{\cdot}49$ cm., $c = 5{\cdot}73$ cm., A $= 65° \ 29'$.

(4) $a = 21{\cdot}43$ cm., $c = 13{\cdot}91$ cm., B $= 52° \ 19'$.

10. In a circle of radius 10 cm., a chord AB subtends an angle of 35° at the centre O. Calculate the area of the triangle OAB.

11. What is the area of a regular hexagon (six sides) inscribed in a circle of radius 10 cm.?

(Find angle subtended by one side at the centre.)

12. Find the area of regular figures of 5, 7, 8, 9 sides inscribed in a circle of radius 10 cm.

13. Find the area of regular figures of 5, 6, 9 sides described with their sides touching a circle of radius 10 cm.

14. The area of a rhombus (parallelogram with equal sides) is 648 square yards and one of its angles is 150°. Find the length of one side.

15. The area of a rhombus is 14·58 sq. cm. and one diagonal is twice as long as the other. Find its angles and sides.

16. The base of an isosceles triangle is 4 cm. long and the area is 20 sq. cm. Find all the angles of the triangle.

17. Find the area of each face and the area of the whole surface of each of the solids given in Exs. 56, 57, Chap. II.

18. A regular hexagon (6 sides) is inscribed in a circle of radius 7 cm. What is the area of the circle? If two adjacent corners A and B be joined to the centre O, what is the area of the *sector* AOB? Find also the area of the *triangle* AOB and hence the area of the segment cut off by AB.

19. Find the area of the segment cut off by one side of a regular pentagon (5 sides) inscribed in a circle of radius 10 cm.

20. If two tangents OA, OB be drawn to a circle of radius 4 cm., and if $\angle AOB = 30°$, what is the area of the figure bounded by the tangents and the larger arc AB?

CHAPTER VIII

LOGARITHMS

1. 10^3 means the product of three tens or $10 \times 10 \times 10$.

What does 10^4 mean? How many tens must be multiplied together to be equal to $10^3 \times 10^4$?

Express $10^3 \times 10^4$ as a power of ten.

What is done to the 3 and 4 to obtain the new index?

Express $10^2 \times 10^7$ and $10^3 \times 10^9$ each as a power of ten

2. To assign a meaning to $10^{0 \cdot 5}$ (or $10^{\frac{1}{2}}$).

["The product of half a ten" has no intelligible meaning.]

What was done with the given indices to obtain the new index in each of the sums in question 1?

Use the same rule to express $10^{\frac{1}{2}} \times 10^{\frac{1}{2}}$ as a power of ten.

Now $4 \times 4 = 16$, and 4 is called the square root of 16.

What is then the meaning and value of $10^{\frac{1}{2}}$ (or $10^{0 \cdot 5}$)?

Remember this result.

3. Express $10^{\frac{1}{3}} \times 10^{\frac{1}{3}} \times 10^{\frac{1}{3}}$ as a power of ten. What is the meaning of $10^{\frac{1}{3}}$? (Compare with question 2.)

What is the meaning and value of $10^{\frac{1}{4}}$ (or $10^{0 \cdot 25}$)?

4. Express $10^{\frac{1}{2}} \times 10^{\frac{1}{4}}$ as a power of ten and find its value.

5. Make a table showing the values of $10^{0.25}$, $10^{0.5}$, $10^{0.75}$, $10^{1.0}$, and use it to draw a graph on squared paper showing the powers of ten which are equivalent to numbers between 1 and 10.

From this graph find the numbers which correspond to $10^{0.3}$, $10^{0.4}$, $10^{0.6}$, $10^{0.85}$. Also make a table showing the powers of ten which correspond to the numbers 2, 3, 4, 5, 6, 7, 8, 9, 10.

The index of the power of ten which is equal to 3 is called the *logarithm* of 3 (to base 10). That is, $\log 3 = 0{\cdot}4771$, for $3 = 10^{0.4771}$; and $\log 10000 = 4$, for $10000 = 10^4$. This should be remembered.

6. Turn to the printed Table of Logarithms and search for the logarithms of the whole numbers from 2 to 10.

[The position of the decimal point is not shown in the table.]

Add a column to the table constructed in question 5, and compare the values obtained from the graph with the values given in the printed Tables.

7. The arrangement of the Table of Logarithms is very similar to that of Tables of Natural Tangents, etc., already familiar.

Now $3{\cdot}5 = 10^{0.5441}$, $3{\cdot}56 = 10^{0.5514}$, $3{\cdot}563 = 10^{0.5518}$. Search for these in the printed Tables. Then write down each of the following and express it as a power of ten by means of the printed Tables:

3·7, 3·75, 3·758, 7·64, 7·649, 9·8, 9·805.

8. Find from the Tables the numbers which are equal to

$$10^{0.3010},\quad 10^{0.8808},\quad 10^{0.4393},\quad 10^{0.4409},\quad 10^{0.4404},$$
$$10^{0.7896},\quad 10^{0.7903},\quad 10^{0.7901},\quad 10^{0.8519},\quad 10^{0.8451}.$$

(These may be found in the Tables of Logarithms or in those of Antilogarithms.)

9. From the Tables $3{\cdot}563 = 10^{0{\cdot}5518}$ and $1{\cdot}212 = 10^{0{\cdot}0835}$.

Therefore $3{\cdot}563 \times 1{\cdot}212 = 10^{0{\cdot}5518} \times 10^{0{\cdot}0835}$.

Now $10^5 \times 10^3 = 10^8$ by adding the indices; thus by the same process

$$3{\cdot}563 \times 1{\cdot}212 = 10^{0{\cdot}5518} \times 10^{0{\cdot}0835} = 10^{0{\cdot}6353}.$$

Write down this line of work and find from the Tables the number whose logarithm is 0·6353; this is the product of $3{\cdot}563 \times 1{\cdot}212$.

Verify by multiplying these two numbers in the ordinary way, but remember that values given in the Tables are not quite exact; they are correct only to four significant figures.

10. Find the value of each of the following by means of logarithms and verify the answer in the usual way:

(1) $2 \times 1{\cdot}427$. (2) $3 \times 2{\cdot}379$. (3) $2{\cdot}5 \times 3{\cdot}624$.

(4) $4 \times 1{\cdot}767$. (5) $3{\cdot}5 \times 2{\cdot}147$.

11. Find by using logarithms the value of each of the following and verify by ordinary multiplication:

(1) $4{\cdot}651 \times 1{\cdot}843$. (2) $3{\cdot}794 \times 2{\cdot}156$.

(3) $1{\cdot}732 \times 1{\cdot}732$. (4) $(2{\cdot}54)^2$.

(5) $7{\cdot}342 \times 1{\cdot}112$. (6) $(3{\cdot}152)^2$.

12. It is clear that $\dfrac{10^5}{10^3} = 10^2$ by *subtracting* the indices.

In the same way $\dfrac{3{\cdot}563}{1{\cdot}212} = \dfrac{10^{0{\cdot}5518}}{10^{0{\cdot}0835}} = 10^{0{\cdot}4683}$.

Write down this line of work and find from the Tables the number whose logarithm is 0·4683. This number is the value of $\dfrac{3{\cdot}563}{1{\cdot}212}$. Verify by long division.

13. Find by logarithms the value of each of the following and verify by division:

(1) $\frac{4\cdot794}{2}$. (2) $\frac{6\cdot315}{5}$. (3) $\frac{9\cdot637}{2\cdot5}$.

(4) $\frac{3\cdot295}{2\cdot34}$. (5) $\frac{7\cdot694}{2\cdot795}$. (6) $\frac{5\cdot407}{1\cdot436}$.

LOGARITHMS OF NUMBERS GREATER THAN TEN

14. Hitherto only numbers between 1 and 10 have been dealt with. Now consider a number greater than 10, for instance 356·3,

$$356\cdot3 = 100 \times 3\cdot563 = 10^2 \times 10^{0\cdot5518} = 10^{2\cdot5518}$$

by adding indices as before.

Write down 35·63, 3563, 35630, 356300, and express each as a power of ten in a similar way.

15. In question 14 what do you observe with regard to the part of each logarithm after the decimal point?

(This part is sometimes called the *mantissa*, which in Latin means "a small part added." The whole number before the decimal point is called the *characteristic*.)

Explain in words why it is that no decimal points are shown in the Table of Logarithms.

16. Write down each of the following and express it as a power of ten. (It is advisable to express each as in the first example of question 14.)

463·4, 79·18, 16·29, 721·4, 72·14,

3114, 76300, 204600, 73·51, 213·5.

17. Find the numbers which are equal to $10^{2\cdot8513}$, $10^{1\cdot5186}$, $10^{3\cdot4843}$, $10^{2\cdot9484}$, $10^{5\cdot4440}$, $10^{3\cdot8522}$, $10^{2\cdot5279}$, $10^{1\cdot4300}$.

18. Find the value of each of the following by logarithms:

(1) $2{\cdot}572 \times 14{\cdot}63$. (2) $31{\cdot}79 \times 43{\cdot}72$.

(3) $727{\cdot}6 \times 1{\cdot}571$. (4) $6297{\cdot} \times 23{\cdot}67$.

(5) $97{\cdot}64 \times 94{\cdot}67$. (6) $212{\cdot}5 \times 409{\cdot}5$.

(7) $21{\cdot}35 \times 37{\cdot}67$. (8) $3{\cdot}244 \times 315{\cdot}4$.

(9) $61{\cdot}12 \times 43150$. (10) 73050×4350.

(11) $\dfrac{14{\cdot}63}{2{\cdot}572}$. (12) $\dfrac{43{\cdot}72}{31{\cdot}79}$.

(13) $\dfrac{727{\cdot}6}{1{\cdot}571}$. (14) $\dfrac{6297}{23{\cdot}67}$.

(15) $\dfrac{57{\cdot}93}{41{\cdot}52}$. (16) $\dfrac{7296}{47}$.

(17) $\dfrac{312400}{76{\cdot}53}$. (18) $\dfrac{49{\cdot}35}{17{\cdot}21}$.

(19) $\dfrac{13{\cdot}69}{13{\cdot}61}$. (20) $\dfrac{200{\cdot}5}{20{\cdot}5}$.

Logarithms of Numbers less than Unity

19. All the numbers treated above are greater than 1. In order to find the logarithm of a number less than 1 (say $0{\cdot}03563$), it is necessary to assign a meaning to expressions such as 10^{-3} (we know that 10^3 means "the product of three tens" but "the product of minus three tens" is not intelligible).

Write down the following in a column and insert their values:

$$10^5 = 100{,}000$$
$$10^4 =$$
$$10^3 =$$
$$10^2 =$$
$$10^1 =$$

Note that the indices on the left-hand side decrease by one each time. How is each of the values on the right-hand side obtained from the one above it? Continue the column downwards,

decreasing the indices as before and find values of 10^0, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}. From this it will be found that $10^{-3}=\frac{1}{1000}$ or $\frac{1}{10^3}$, and this has an intelligible meaning. **Remember this result.**

20. Now to find the logarithm of 0·03563 we have

$$0{\cdot}03563=\frac{3{\cdot}563}{100}=\frac{1}{100}\times 3{\cdot}563=10^{-2}\times 10^{0\cdot5518}=10^{-2+0\cdot5518}$$ by adding indices as before, and this is written $10^{\bar{2}\cdot5518}$ to show that the minus sign applies to the 2 only. This arrangement is adopted in order to simplify the Tables, as will be seen hereafter.

By writing each of the following in the same way as in question 20 above, express it as a power of ten: 0·01212, 0·1212, 0·3563, 0·0003563.

21. Write down the following in a column and express each as a power of ten: 3563, 356·3, 35·63, 3·563, 0·3563, 0·03563, 0·003563, 0·0003563. By adopting the system of writing the minus sign over the characteristic (*e.g.* $\bar{2}{\cdot}5518$), it is found that the same Table of Logarithms may be used for numbers greater than or less than 1.

Discover and express in words a rule connecting the characteristic of the logarithm with the position of the decimal point of the number it represents.

22. Express each of the following as a power of ten:

0·0345, 0·7691, 0·00463, 0·0001105,
0·9971, 0·9065, 0·0435, 0·5432.

Note. To find the value of 0·02 × 0·007 we have

$$\begin{aligned} 0{\cdot}02\times0{\cdot}007 &= 10^{\bar{2}\cdot3010}\times10^{\bar{3}\cdot8451} \\ &= 10^{\bar{4}\cdot1461} \\ &= 0{\cdot}00014. \end{aligned} \qquad \begin{array}{r} \bar{2}{\cdot}3010 \\ \bar{3}{\cdot}8451 \\ \hline \bar{4}{\cdot}1461 \end{array}$$

The addition is done in the usual way, but it must be remembered that $\bar{2}$ and $\bar{3}$ are simply −2 and −3.

23. Find the value of each of the following products by logarithms and verify each one:

(1) $0{\cdot}5 \times 40$. (2) $0{\cdot}004 \times 50$. (3) $0{\cdot}025 \times 160$.

(4) $0{\cdot}5 \times 0{\cdot}5$. (5) $(0{\cdot}04)^2$. (6) $0{\cdot}04 \times 0{\cdot}05$.

(7) $0{\cdot}012 \times 0{\cdot}2$. (8) $0{\cdot}0008 \times 40$. (9) $0{\cdot}0011 \times 0{\cdot}02$.

(10) $0{\cdot}25 \times 0{\cdot}0003$.

24. Find the value of the following by logarithms:

(1) $0{\cdot}02572 \times 0{\cdot}1463$. (2) $0{\cdot}02367 \times 0{\cdot}6297$.

(3) $0{\cdot}7276 \times 0{\cdot}01571$. (4) $0{\cdot}4372 \times 0{\cdot}003179$.

(5) $946{\cdot}7 \times 0{\cdot}09764$. (6) $3{\cdot}244 \times 0{\cdot}3154$.

Note. To find the value of $0{\cdot}2 \div 0{\cdot}007$ we have

$$\frac{0{\cdot}2}{0{\cdot}0007} = \frac{10^{\bar{1}\cdot 3010}}{10^{\bar{3}\cdot 8451}} = 10^{1\cdot 4559} = 28{\cdot}57.$$

The subtraction is best effected by dealing with the $\bar{3}$ first.

The operation

$$\begin{array}{r} \bar{1}{\cdot}3010 \\ \bar{3}{\cdot}8451 \\ \hline 1{\cdot}4559 \end{array}$$

is then the same as

$$\begin{array}{r} 2{\cdot}3010 \\ 0{\cdot}8451 \\ \hline 1{\cdot}4559 \end{array}$$

for $-\bar{3}$ is the same as $+3$. Check by adding the last two numbers.

25. Find the value of each of the following by logarithms:

(1) $\dfrac{0{\cdot}001463}{0{\cdot}2572}$. (2) $\dfrac{0{\cdot}7276}{0{\cdot}01571}$. (3) $\dfrac{0{\cdot}002367}{0{\cdot}6297}$

(4) $\dfrac{0{\cdot}4372}{0{\cdot}003179}$. (5) $\dfrac{0{\cdot}09764}{0{\cdot}9467}$. (6) $\dfrac{3{\cdot}244}{0{\cdot}3154}$

26. In questions 2 and 3 of this Chapter it was found that $10^{\frac{1}{2}}$ meant $\sqrt{10}$ and $10^{\frac{1}{3}}$ meant $\sqrt[3]{10}$.

In the same way $\sqrt[3]{50} = (50)^{\frac{1}{3}} = (10^{1\cdot 6990})^{\frac{1}{3}}$.

Now $(10^4)^3 = 10^4 \times 10^4 \times 10^4 = 10^{12}$, the new index being obtained by *multiplying* 4 by 3.

In the same way $(10^{1\cdot 6990})^{\frac{1}{3}} = 10^{0\cdot 5663}$ by *multiplying* 1·6990 by $\frac{1}{3}$, *i.e.* by dividing it by 3.

Find the number whose logarithm is 0·5663. It is $\sqrt[3]{50}$.

27. In a similar manner find the values of

$$\sqrt{320}, \qquad \sqrt[3]{76}, \qquad \sqrt[3]{1140}, \qquad \sqrt{2491}, \qquad \sqrt[4]{1500}.$$

28. In dealing with the logarithm of a number less than 1, remember that the decimal part (or mantissa) must always be kept positive, the characteristic being negative.

$$\sqrt{\cdot 002} = (\cdot 002)^{\frac{1}{2}} = (10^{\bar{3}\cdot 3010})^{\frac{1}{2}}.$$

What is half of $\bar{3}\cdot 3010$ (*i.e.* $-3 + 0\cdot 3010$)?

To transform the answer so that the decimal part remains positive, it is most convenient to think of $\bar{3}\cdot 3010$ as being equal to $\bar{4} + 1\cdot 3010$. Dividing this by 2, the logarithm of $\sqrt{\cdot 002}$ may be found, and hence its value from the Tables.

29. In a similar manner find by logarithms the value of the following, verifying the answers:

(1) $\sqrt{0\cdot 04}$. (2) $\sqrt{0\cdot 004}$. (3) $\sqrt{0\cdot 0004}$.

(4) $\sqrt[3]{0\cdot 008}$. (5) $\sqrt[3]{0\cdot 0008}$. (6) $\sqrt[3]{0\cdot 08}$.

(7) $\sqrt{0\cdot 765}$. (8) $\sqrt[3]{0\cdot 057}$. (9) $\sqrt[4]{0\cdot 3761}$.

(10) $\sqrt[4]{0\cdot 0015}$.

Miscellaneous Examples on Logarithms.

When the principles have been grasped it is convenient to arrange the work as in the following Examples.

A. $23{\cdot}57 \times 3{\cdot}721 = 87{\cdot}70.$

No.	log
23·57	1·3724
3·721	0·5706
	1·9430

B. $\dfrac{372{\cdot}1}{2{\cdot}357} = 157{\cdot}9.$

No.	log
372·1	2·5706
2·357	0·3724
	2·1982

Find the following:

(1) $72{\cdot}47 \times 26{\cdot}05.$

(2) $631{\cdot}4 \times 0{\cdot}05641.$

(3) $\dfrac{23{\cdot}57}{3{\cdot}461}.$

(4) $\dfrac{326{\cdot}4}{435{\cdot}6}.$

(5) $(17{\cdot}49)^2.$

(6) $(31{\cdot}62)^3.$

(7) $\sqrt{29{\cdot}95}.$

(8) $\sqrt[3]{12}.$

(9) $(0{\cdot}7462)^4.$

(10) $(0{\cdot}06925)^3.$

(11) $\sqrt{0{\cdot}007534}.$

(12) $\sqrt[3]{0{\cdot}04691}.$

(13) $\sqrt{(0{\cdot}1193)^3}.$

(14) $\sqrt[3]{(0{\cdot}0072)^2}.$

(15) $\dfrac{0{\cdot}3157}{0{\cdot}04694}.$

(16) $\left(\dfrac{73{\cdot}91}{0{\cdot}947}\right)^2.$

(17) $\sqrt[3]{\dfrac{0{\cdot}0724}{7{\cdot}4 \times 3{\cdot}142}}.$

(18) $\dfrac{7{\cdot}529 \times 0{\cdot}04392}{0{\cdot}5497}.$

(19) $\dfrac{3{\cdot}142}{0{\cdot}375 \times (0{\cdot}074)^2}.$

(20) $\dfrac{(0{\cdot}325)^3}{0{\cdot}247 \times (0{\cdot}04627)^2}.$

Note. In working out an example such as $\frac{57 \times \sin 69^\circ}{143}$ it is convenient to make a skeleton arrangement before looking out logs from the Tables, thus :—

57 sin 69°	
143	

The logarithm of sin 69° may be found direct from the Table of Logarithmic Sines; in some Tables 10 is added to the characteristic to avoid the clumsy printing of minus signs above the numbers. The work then stands thus :—

$$\frac{57 \times \sin 69^\circ}{143} = 0{\cdot}3722.$$

57 sin 69°	1·7559 $\bar{1}$·9702
143	1·7261 2·1553
	$\bar{1}$·5708

Find the value of :

(21) 16 sin 35°.

(22) 243 cos 47°.

(23) 11·61 tan 49°.

(24) 63 sin 17° 24′.

(25) 114 tan 19° 24′.

(26) 1·64 cos 23° 47′.

(27) $13{\cdot}41 \times 7{\cdot}61 \times \cos 14^\circ\ 19'$.

(28) $\frac{12{\cdot}6}{15{\cdot}5} \sin 41^\circ\ 37'$.

(29) $73{\cdot}56 \cos 19^\circ\ 21' + 61{\cdot}42 \sin 25^\circ\ 52'$.

(30) $\frac{61 \sin 49^\circ}{95}$.

(31) $\frac{13 \sin 29^\circ\ 46'}{14}$.

(32) $12{\cdot}6 \times \frac{\sin 43^\circ\ 17'}{\sin 32^\circ\ 24'}$.

(33) $2 \times 3{\cdot}24 \times 5{\cdot}62 \cos 72^\circ\ 31'$.

(34) $2 \times 7{\cdot}63 \times 8{\cdot}5 \times \cos 31^\circ\ 37'$.

(35) $\frac{26{\cdot}49}{25{\cdot}7} \sin 23^\circ\ 14'$.

(36) $\frac{19{\cdot}43 \sin 73^\circ\ 29'}{10{\cdot}65}$.

CHAPTER IX

SOLUTION OF TRIANGLES WHICH ARE NOT RIGHT-ANGLED

Note. The calculations in this and the following chapters may often be much simplified by the use of logarithms.

A. *Given two sides and the angle included.*

1. Draw accurately a triangle ABC having angle $A = 60°$, $b = 10$ cm. and $c = 9$ cm. Measure B, C and a.

2. If, in the triangle above, CN is drawn perpendicular to AB, the right-angled triangle ACN can be solved by calculation, and then the parts of the triangle BCN can also be calculated.

Calculate B, C, a by this means.

3. Solve the same triangle by drawing a perpendicular from B to AC.

4. Draw a freehand sketch of the triangle ABC in which $A = 53° \, 8'$, $b = 7{\cdot}5$ cm., $c = 9$ cm. Solve it by calculation and verify by accurate drawing.

5. Repeat for a triangle having $A = 23° \, 35'$, $b = 10$ cm. and $c = 12$ cm.

6. In the questions 1, 2, 3 above it was found possible to solve the triangle by drawing a perpendicular from C or from B. Is it possible to do so by drawing a perpendicular from A? Express *very carefully in words* the line (or lines) which should be drawn in order to solve a triangle in which two sides and the included angle are given.

7. Solve the following triangles and verify the answers by drawing to scale.

Note. An example of orderly arrangement will be found in the Appendix.

(1) $a = 25$, $b = 24$, $C = 41^\circ\ 25'$.

(2) $a = 2\cdot6$, $c = 2\cdot3$, $B = 61^\circ\ 24'$.

(3) $b = 44\cdot7$, $c = 46\cdot9$, $A = 24^\circ\ 17'$.

8. If $A = 60^\circ$, $b = 10$ cm., $c = 3$ cm., solve by drawing a perpendicular from B to AC.

9. Solve the triangle of question 8, by drawing CN perpendicular to AB *produced* and dealing with the right-angled triangles ACN and BCN.

10. Solve the triangle in which $A = 126^\circ\ 52'$, $b = 10$ cm., $c = 7\cdot5$ cm. by drawing the perpendicular from B to AC produced.

11. Is it possible to solve the last triangle by drawing a perpendicular (1) from A, (2) from B?

If either of these is possible, complete the solution.

12. Solve the following triangles by drawing a perpendicular:

(1) $b = 23\cdot4$, $c = 11\cdot6$, $A = 37^\circ\ 49'$.

(2) $a = 149$, $b = 51$, $C = 58^\circ\ 23'$.

(3) $a = 26\cdot4$, $b = 31\cdot7$, $C = 135^\circ\ 16'$.

(4) $b = 3\cdot49$, $c = 4\cdot37$, $A = 98^\circ\ 23'$.

(5) $c = 0\cdot45$, $a = 0\cdot19$, $B = 161^\circ\ 47'$.

B. *Given two angles and one side.*

1. Draw as accurately as possible a triangle ABC in which $A = 53^\circ\ 8'$, $B = 59^\circ\ 29'$ and $c = 10$ cm. Measure C, a and b.

2. From B drop a perpendicular BN to AC and *calculate* C, AN, NB, NC, CB and CA in this order.

3. Draw a sketch of the triangle given above and write down *how* it can be solved if a perpendicular is drawn from A instead of B. Indicate the parts to be found in their proper order as in question 2.

4. Is it possible to solve the triangle by drawing a perpendicular from C? Describe *in words* the line (or lines) which should be drawn to solve a triangle when two angles and the side between them are given.

5. If two angles and the side opposite to one of them (A, B, a) be given, from which angle should a perpendicular be drawn in order to solve the triangle?

Is it possible to solve it in more than one way? Indicate how the triangle should be solved.

6. Solve by this method the triangles in which

(1) $A = 43° 27'$, $B = 61° 9'$, $a = 3{\cdot}5$.

(2) $A = 71° 42'$, $B = 65° 19'$, $c = 16{\cdot}9$.

(3) $A = 29° 17'$, $B = 108° 53'$, $b = 214$.

(4) $A = 13° 39'$, $C = 41° 11'$, $a = 14{\cdot}63$.

(5) $C = 112° 41'$, $A = 47° 25'$, $b = 29{\cdot}46$.

C. *Given three sides.*

1. Draw a sketch of the triangle ABC in which $a = 5$ cm., $b = 6$ cm. and $c = 4$ cm. Draw AN perpendicular to BC. Let $BN = x$ cm. What is then the length of CN?

Using Pythagoras' Theorem in the triangles ABN, ACN, find two expressions for the square on AN and by equating them find the length x. Complete the solution of the triangle ABC and verify by drawing an accurate figure.

2. Verify the results of question 1, by drawing the perpendicular from C and solving as before.

3. Draw the perpendicular from B in the same triangle and indicate (without actual calculation but by writing down the parts to be found in their proper order) *how* the triangle may be solved.

4. Solve the following triangles. Consider carefully which perpendicular will lead to the simplest calculation:

(1)	$a = 7$ cm.,	$b = 8$ cm.,	$c = 10$ cm.
(2)	$a = 4$,	$b = 3$,	$c = 2$.
(3)	$a = 2{\cdot}5$,	$b = 3$,	$c = 1$.
(4)	$a = 2{\cdot}5$,	$b = 3{\cdot}5$,	$c = 4$.
(5)	$a = 6{\cdot}34$,	$b = 7{\cdot}42$,	$c = 3{\cdot}5$.

D. *Given two sides and an angle opposite one of them.*

1. Draw as accurately as possible a triangle having

$$\text{A} = 36^\circ\ 52', \quad b = 10 \text{ cm.}, \quad a = 8 \text{ cm.}$$

Measure B, C and c; there are *two* answers to each.

2. Draw CN perpendicular to AB in the triangle above and *calculate* the values of B, C and c by solving the triangles ACN and BCN.

3. Is it possible to solve the same triangle by drawing the perpendicular (1) from B, or (2) from A? Illustrate the answer with a sketch.

4. Are there two possible values for c in a triangle having $\text{A} = 36^\circ\ 52'$, $b = 10$ cm. and $a = 12$ cm.?

Solve this triangle completely.

5. In the triangle of question 1, a is *less* than b, and in question 4, a is *greater* than b. Express *clearly in words* the conditions under which two different solutions of a triangle may be expected when two sides and an angle opposite one of them are given. Illustrate with sketches.

E. *Miscellaneous examples on solution by dividing into two right-angled triangles.*

1. There are six parts of a triangle—three sides and three angles. When three of these are known it is generally possible to calculate the other three and solve the triangle.

The list below gives the possible ways in which three of these parts may be given.

Copy each item and determine by drawing a sketch (1) whether the triangle can be solved, and if so (2) the first step in the solution:

(i) 3 sides (a, b, c).

(ii) 2 sides and the included angle (a, b, C).

(iii) 2 sides and an angle not included (a, b, A).

(iv) 2 angles and the side included (A, B, c).

(v) 2 angles and a side not included (A, B, a).

(vi) 3 angles.

In which cases may two different solutions possibly occur?

2. Solve the following triangles by drawing a perpendicular:

(1)	$A = 54° 23'$,	$b = 19{\cdot}45$,	$c = 23{\cdot}51$.
(2)	$A = 117° 21'$,	$B = 35° 24'$,	$c = 4{\cdot}67$.
(3)	$A = 41° 41'$,	$b = 26{\cdot}7$,	$a = 20{\cdot}63$.
(4)	$a = 4{\cdot}35$,	$b = 7{\cdot}91$,	$c = 6{\cdot}41$.
(5)	$B = 63° 26'$,	$b = 114$,	$a = 91{\cdot}5$.
(6)	$B = 24° 8'$,	$C = 79° 53'$,	$a = 3{\cdot}47$.
(7)	$C = 47° 19'$,	$a = 191$,	$b = 179$.
(8)	$a = 13{\cdot}5$,	$b = 14$,	$c = 16{\cdot}5$.
(9)	$C = 37° 23'$,	$c = 91$,	$a = 124$.
(10)	$C = 71° 35'$,	$A = 52° 13'$,	$b = 7{\cdot}47$.
(11)	$a = 147$,	$b = 92$,	$c = 69$.
(12)	$A = 51° 29'$,	$B = 79° 47'$,	$a = 12{\cdot}39$.

Note. The problems in Chapter XIII may all be solved by the process above, but the general formulae in Chapters X, XI, XII will be found to shorten the work considerably.

CHAPTER X

OBTUSE ANGLES

1. In the figure P revolves round the circle whose centre is O and whose radius is 10 cm. AO = 17 cm. PN is drawn perpendicular to AOQ.

Calculate the lengths of ON and AN when the angle POQ has the values: 36° 52′, 53° 8′, 60°, 120°, 126° 52′, 143° 8′ and arrange the results in a table.

It will be seen that when POQ is obtuse, AN is *less* than 17 cm.

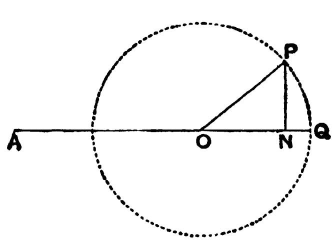


Fig. 22.

But in the figure as drawn, AN = 17 + 10 cos POQ. If this is also true when POQ is obtuse, what do you conclude with regard to the value of the cosine of an obtuse angle?

2. For which pairs of the angles above is the *length* of ON the same? What is the sum of each pair? Write down the values of cos 120°, cos 126° 52′, cos 143° 8′ and express in words the relation between the cosines of supplementary angles.

3. Find the length of PN for each of the angles in question 1. For which pairs is the length of PN the same? It is customary to say that PN = 10 sin POQ for all values of the angle POQ acute or obtuse.

Write down the values of sin 120°, sin 126° 52′, sin 143° 8′.

Note that PN is drawn *above* the line in every case, but ON is drawn from O in one direction for an acute angle POQ, but in the *opposite direction* for an obtuse angle POQ.

4. The figure shows that $\cos 112° 37' = -\frac{5}{13}$. Calculate the length of the perpendicular (Pythag.), and write down the values of the other five ratios of this angle, paying special attention to *sign*.

5. From the Tables find the values of the *sine* and *cosine* of: 140°, 123°, 104° 13′, 166° 51′, 152° 22′.

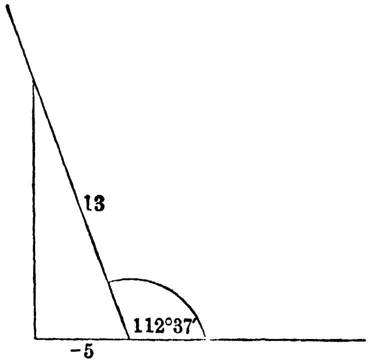


Fig. 23.

6. Find the *tangent* of each of these angles. Draw a rough sketch to determine the sign.

7. Find the *cotangent*, *secant* and *cosecant* of each of the angles above.

8. Find the value of

sin 64°,	sin 164°,	cos 53° 30′,	cos 153° 30′,
sin 101° 25′,	cos 121° 33′,	sin 111° 11′,	cos 174° 19′,
tan 41° 23′,	tan 141° 23′,	cot 19° 21′,	cot 99° 21′,
cosec 31° 41′,	sec 12° 17′,	sec 131° 41′,	cosec 112° 17′.

9. Find *two* angles less than 180°, whose sine is 0·7660.

10. If A is an angle of a triangle, find A if

$\sin A = 0·9$.	$\cos A = -0·6225$,	$\sin A = 0·2860$,
$\cos A = 0·1225$,	$\cos A = -0·1225$,	$\sin A = 0·3333$,
$\tan A = 2·6051$,	$\tan A = -2·6051$,	$\cot A = -0·2401$,
$\operatorname{cosec} A = 1·1383$,	$\sec A = 3·1085$,	$\sec A = -3·1085$.

11. In dealing with triangles any angle between 0° and 180° may occur. What can be asserted with regard to the sine of any angle of a triangle? What can be asserted with regard to the cosine of any angle of a triangle? These results should be carefully remembered.

CHAPTER XI

THE SINE FORMULA

1. In the triangle ABC having $A = 40°$, $B = 80°$ and $a = 10$ cm. calculate C, b and c by drawing a perpendicular, and show that

$$\frac{b}{a} = \frac{\sin B}{\sin A}.$$

2. Show also that $\frac{c}{a} = \frac{\sin C}{\sin A}$ and $\frac{b}{c} = \frac{\sin B}{\sin C}$ in this particular triangle.

3. Draw any acute-angled triangle at random (*i.e.* without making the angles and sides any chosen size or length). Measure the sides and angles and show that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

4. Explain *clearly in words* the facts verified above.

5. Consider whether the rule is true for obtuse-angled triangles. Verify by drawing such a triangle at random and carefully measuring its sides and angles. (Remember the fact with regard to the sines of supplementary angles which was dealt with in Chapter X.)

6. Making use of the sine rule above, calculate a in the triangle having $A = 62°$, $C = 53° 8'$, $c = 8$ cm., and verify by accurate drawing.

7. Calculate the following and verify by accurate drawing.

Note. It is convenient to write the required part as the numerator of a fraction.

(1) $A = 63°$, $C = 50°$, $c = 6$ cm., find a.

(2) $B = 47°$, $C = 73°$, $c = 11$ cm., find b.

(3) $C = 30°$, $A = 81° 30'$, $a = 6$ cm., find c.

(4) $A = 44°$, $B = 31°$, $a = 4$ cm., find b.

(5) $B = 39° 24'$, $C = 49° 12'$, $b = 5$ cm., find c.

8. If $10 \sin A = 3 \sin B$, what is the value of $\dfrac{\sin A}{\sin B}$?

And if $\dfrac{c}{a} = \dfrac{\sin C}{\sin A}$, show *clearly* that $\dfrac{a}{\sin A} = \dfrac{c}{\sin C}$.

9. To *prove* the Sine Formula for an acute-angled triangle draw a sketch of *any* such triangle ABC. Draw CN perpendicular to AB. In the triangle CAN express CN in terms of b and A; find also an expression for CN in terms of a and B in the triangle BAN. By equating these prove that $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$ for any acute-angled triangle.

Note. An alternative proof will be found in the Appendix.

10. Remembering that $\sin(180° - A) = \sin A$, prove the sine formula for any obtuse-angled triangle.

11. Use the Sine Rule to find the following:

(1) If $A = 36° 52'$, $B = 30°$, $a = 10$ cm., find b.

(2) If $A = 58° 13'$, $C = 23° 35'$, $c = 4$ cm., find a.

(3) If $B = 68° 26'$, $C = 53° 8'$, $c = 3{\cdot}5$ cm., find b.

Note. Much time may be saved by *saying* "a over b equals sin A over sin B" and at the same time *writing* $a = b \times \dfrac{\sin A}{\sin B}$.

(4) If $A = 51^\circ 16'$, $C = 73^\circ 49'$, $a = 12$ cm., find c.

(5) If $B = 20^\circ 11'$, $A = 59^\circ 51'$, $b = 17$ cm., find a.

(6) If $A = 43^\circ 32'$, $B = 61^\circ 39'$, $c = 10$ cm., find C and a.

(7) If $B = 21^\circ 13'$, $C = 51^\circ 42'$, $a = 4$ cm., find b.

(8) If $A = 44^\circ 32'$, $C = 67^\circ 21'$, $b = 5$ cm., find a.

(9) If $A = 21^\circ 6'$, $b = 10$, $a = 6$, find two values for B.

(10) If $C = 32^\circ 37'$, $c = 3{\cdot}6$, $b = 5$, find two values for B.

(11) If $B = 37^\circ 23'$, $b = 2$, $a = 2{\cdot}5$, find A. Are there two values?

(12) If $B = 37^\circ 23'$, $b = 3$, $a = 2{\cdot}5$, find A. Are there two values?

12. If in Fig. 1, p. 2 the angle $A = 24^\circ$, $\angle BPC = 54^\circ$ and $AP = 10$ cm., calculate $\angle ABP$, BP (by the sine formula) and hence find BC.

13. In the same fig. if $A = 41^\circ$, $\angle BPC = 61^\circ$ and $AP = 200$ feet find a trigonometrical expression for BC *without working with tables.* By substituting the values given in the tables calculate the length of BC.

14. Use the method indicated above to solve the following. The angle of elevation of the top of a tower is observed to be 25° and from a point 100 feet nearer it is 43°. Find the height of the tower.

15. From a boat the angle of elevation of the top of a cliff 200 feet high is found to be 29°. The boat sails directly towards the cliff. How many feet must it sail so that the angle of elevation may be 50°?

16. From the top of a cliff the angles of depression of two buoys in a straight line and 124 feet apart are 18° 47′ and 51° 29′. How high is the cliff?

Note. Questions B 6 and several in E 2 of Chapter IX may be solved by means of the sine formula.

CHAPTER XII

THE COSINE FORMULA

1. In the triangle ABC if $b = 3$ in., $c = 4$ in. and $A = 90°$, show that a must be 5 in. long.

If A is acute, will a be greater or less than 5 in.?

If A is obtuse, will a be greater or less than 5 in.?

To find a relation between the length of a and the size of the angle A in this triangle, draw an accurate figure with $A = 20°$, $40°$, $60°$, $80°$ in succession, and measure the length of a in each case. Verify that $b^2 + c^2 - a^2 = 2bc \cos A$ in each case. Arrange results in a table.

2. By drawing a perpendicular CN from C to AB in each case above and *calculating* the lengths of AN, CN, NB, a, verify that $b^2 + c^2 - a^2 = 2bc \cos A$.

3. In Chapter IX, question A 2, it was found that if $b = 10$ cm., $c = 9$ cm. and $A = 60°$, then $a = 9{\cdot}539$ cm.

Verify the formula above by substituting these values.

Note. This formula is very useful and should be remembered. A formal proof will be found in the Appendix.

4. Verify the following by means of the Cosine Formula:

(1) If $b = 7{\cdot}5$ cm., $c = 9$ cm., $A = 53° \, 8'$, then $a = 7{\cdot}5$ cm.

(2) If $b = 10$ cm., $c = 12$ cm., $A = 23° \, 35'$, then $a = 4{\cdot}903$ cm.

(3) If $b = 25$ cm., $c = 24$ cm., $A = 41° \, 25'$, then $a = 17{\cdot}35$ cm.

5. If in any one of the figures above the letters denoting the sides and angles were altered so that A became B, B became C, a became b, etc., then what relation between the sides and cos B would be true?

Write down the cosine formula in a triangle ABC in three different forms, thus:

$$a^2 = b^2 + c^2 - \dots, \quad b^2 = \dots\dots\dots\dots \cos B,$$
$$c^2 = \dots\dots\dots\dots$$

6. If the angle A is a right angle, what is the value of cos A, and what does the formula become?

Observing that $\cos B = \frac{c}{a}$ in this triangle, show that the formula is true in each of the three forms given in question 5.

7. Consider whether the same formula is true in an obtuse-angled triangle.

To do this calculate the length of a in a triangle ABC having $b = 8$ cm. and $c = 10$ cm. when A is 90°, 110°, 130°, 150°, 170°. (Draw CN perpendicular to AB produced.) Arrange results in a table and (remembering the important fact about the cosine of an obtuse angle) verify that $a^2 = b^2 + c^2 - 2bc \cos A$.

8. Verify the cosine formula in each of the following:

(1) If $b = 10$ cm., $c = 7{\cdot}5$ cm. and $A = 126° 52'$, then $a = 15{\cdot}69$ cm.

(2) If $a = 19$ cm., $c = 45$ cm. and $B = 161° 47'$, then $b = 63{\cdot}33$ cm.

9. Find the third side in each of the following triangles:

(1) $a = 5$, $b = 4$, $C = 32°$.

(2) $a = 5$, $b = 4$, $C = 148°$.

(3) $b = 3{\cdot}5$, $c = 10$, $A = 71° 36'$.

(4) $b = 3{\cdot}5$, $c = 10$, $A = 108° 24'$.

(5) $a = 6{\cdot}25$, $c = 3{\cdot}2$, $B = 21° 13'$.

(6) $a = 17{\cdot}3$, $b = 23{\cdot}4$, $C = 34° 25'$.

(7) $a = 17{\cdot}3$, $b = 23{\cdot}4$, $C = 145° 35'$.

10. By substituting in the cosine formula, find the angles of the following triangles. Verify by accurate drawing.

(1) $a = 4$ in., $b = 5$ in., $c = 6$ in.

(2) $a = 5$ in., $b = 6$ in., $c = 7$ in.

(3) $a = 4$ in., $b = 5$ in., $c = 3$ in.

(4) $a = 7 \cdot 5$ cm., $b = 4 \cdot 4$ cm., $c = 4 \cdot 3$ cm.

11. If the sides of a triangle are 3, 5 and 6 inches long, calculate the smallest angle.

12. Find the largest angle in the triangle whose sides are 4, 5, 2 inches long.

13. Repeat question E 1, Chap. IX, showing how each of the required parts may be found by using either the cosine formula or the sine formula.

14. Two trains start at the same time from the same station along straight tracks making an angle of 40°. If their average speeds are 20 and 24 miles an hour, how far apart are they in half an hour?

15. What angle does a straight rod 29 feet long subtend at the eye of an observer who is 18 feet from one end and 25 feet from the other?

Note. Questions C 4 and several of those in E 2 of Chapter IX may be solved by means of the cosine formula.

CHAPTER XIII

PROBLEMS

Note. An example of orderly arrangement will be found in the Appendix.

1. From the ends of a breakwater a mile long and running E. and W. the bearings of a certain buoy are found to be S. 27° W. and S. 72° E. Find the distance of the buoy from each end in yards, and also its distance from the nearest point on the breakwater.

2. From the same breakwater the bearings of another buoy are S. 63° 53′ W. and S. 41° 19′ W. Find its distance from each end.

3. From the E. end of the same breakwater a point on shore is found to be 1500 yds. N. 15° 28′ W. Find the distance of this point from the other end.

4. If A is 6·4 miles S. 17° 35′ E. of P, and B is 4·3 miles S. 31° 17′ W. of P, what is the distance AB?

5. A balloon is observed from two points 1250 feet apart on a level plain at the moment when it passes above the line joining them. If the angles of elevation are found to be 39° 51′ and 61° 23′, find the distance of the balloon from each point and its height above the plain.

6. A vertical wall runs horizontally across the side of a hill which slopes at an angle of 10° 25′ with the horizontal. From a point down the slope 110 feet from the base of the wall the angle of elevation of the top is found to be 15° 57′. Find the height of the wall.

7. The angles of elevation of the top of a certain tree from two points on opposite sides of it are found to be 52° 31′ and 63° 5′. If the points are 125 feet apart, find the height of the tree.

8. In a certain steam engine the crank OP is 18 inches long and revolves about O. The other end C of the connecting rod CP (which is 72 inches long) moves backwards and forwards along a straight line passing through O. Find the angle at C and the distance CO when the angle COP is

(1) 24°, (2) 90°, (3) 124°.

9. Find the angle at O and the distance CO in the same engine when the angle at C is (1) 13°, (2) 10° 23′. (There are two answers to each.)

10. A tower stands on a slope which is inclined at an angle of 17° 15′ with the horizontal. From a point further up the slope and 370 feet from the base of the tower the angle of depression of the top of the tower is found to be 9° 38′. Find the height of the tower.

11. From a ship sailing N. $22\frac{1}{2}$° W. a lighthouse bears N. 31·3° E. After the ship has sailed 12 miles the lighthouse is found to bear due E. Find its distance from the ship at each observation.

12. P and Q are points at opposite ends of a wood. From a point R outside the wood the angle PRQ is observed to be 79° 47′, and PR = 272 yds. and QR = 349 yds. Find the distance PQ in yards.

13. In the triangle ABC having B = 49° 52′, C = 61° 34′, a = 17·4 cm., the bisectors of the angles meet at I. Find the radius r of the circle which touches AB, BC and CA.

(If ID is perpendicular to BC, it is a radius. Find BD in terms of r, CD in terms of r.)

14. In the triangle ABC, A = 43°, a = 12·5 cm. What angle does BC subtend at the centre of the circle passing through A, B, C? Calculate the radius of the circle. If b = 9·7 cm., find the angle which it subtends at the centre.

15. A tower subtends an angle of 34° at a point P which is 80 yards due East of it. What angle will it subtend at a point Q which is 60 yards due South of P?

16. A ladder 20 feet long makes an angle of 30° with the wall of a house. How much nearer to the wall must the foot of the ladder be brought in order to reach 18 inches higher than before, and what angle will the ladder now make with the wall?

17. A vessel starts from O, sails 4 miles N. 38° E. to P, and then 6 miles S. 16° E. to Q. She then returns direct to O. How far and in what direction has she to go, and how near will she pass to P?

18. From the top of a wall 5 feet high the angle of elevation of the top of a flagstaff is observed to be 63° 31′, while the angle of depression of the foot is 5° 42′.

Find (*a*) the distance of the foot of the wall from the foot of the flagstaff, (*b*) the height of the flagstaff.

19. A straight road runs N.E. for 300 yards from A to B. From A a tower bears N. 19° 27′ E., and from B, N. 70° 33′ W. Find the distance of the tower from A.

20. Solve the triangle in which $a = 28{\cdot}3$ cm., $c = 12{\cdot}5$ cm. and $B = 32°$.

21. In a triangle $B = 40°$, $C = 70°$, $a = 123$ yards. Find the side c and also the area of the triangle.

22. In a triangle $a = 5$ in., $b = 7$ in., $c = 9$ in. Find the greatest angle.

23. The length of a breakwater which lies along a meridian is 789 yards. From a boat the Southern extremity bore S. 67° 30′ W. and its distance was 426 yards.

Find (i) the distance of the boat from the breakwater, (ii) the bearing and distance of the boat from the Northern end of the breakwater.

24. Two ships leave a port at the same instant. One steams S. 11° 15′ E. at 15 knots, and the other S. 22° 30′ W. at 19 knots. Find their distance apart at the end of half an hour and the bearing of one from the other.

25. A parallelogram ABCD has $AB = 3{\cdot}64''$, $BC = 5{\cdot}82''$, and the angle $B = 67°$. Calculate the length of the perpendicular drawn from A upon BC and also the angles which the diagonal AC makes with the sides of the parallelogram.

26. Two tangents to a circle of radius 5 cm. from an external point are each 8 cm. long. What length of arc is included between the tangents?

27. The sides of a parallelogram are 12″ and 8″, and include an angle of 37° 32′. Find the area of the parallelogram, and the lengths of its diagonals.

28. A ship S at sea finds the bearings of a lighthouse L and a buoy B to be N. 32° W. and N. 14° E. respectively. B is found to be 3·5 miles N. 66° E. from L. Find the distances of the ship from the lighthouse and the buoy.

29. A ladder reaches a window ledge 26 feet above the ground on one side of a street, and makes 70° with the ground. Find the length of the ladder.

On turning the ladder over without moving its foot, it is found that when it rests against a wall on the opposite side of the street it makes 20° with the ground. Find the width of the street.

30. Two lighthouses are known to be $8\frac{3}{4}$ miles apart. A ship observes their bearings to be S. 41° 48′ W. and S. 19° 7′ W. respectively. After making due South for some time she observes that they are both in a straight line due West. How far has she sailed?

31. A, B are two points 1200 feet apart on the straight bank of a river flowing due East. A point C on the opposite bank bears N. 53° 19′ E. from B and N. $24\frac{1}{2}$° W. from A. Find the perpendicular width of the river at C.

32. In a circle of radius 3 cm. an isosceles triangle ABC is described with base BC 3·5 cm. long. Solve the triangle ABC.

33. The diagonals of a rhombus are 13 cm. and 9 cm. long. Calculate the length of the radius of the circle inscribed in the rhombus.

34. For a quick survey of a bay a ship observed two points P and Q at sea level.

Bearing of P, S. 73° W.

" " Q, S. 87° E.

At P angle of elevation of masthead is 3°.

" Q " " " " " 5°.

Height of masthead above water-line 120′.

Find (i) distance of P from ship, in yards;

(ii) length of base line PQ, in yards.

35. (i) Solve a triangle ABC, given AB = 23·6 in., BC = 18·5 in., angle A = 28°. Are there two possible solutions?

(ii) Solve a triangle ABC, given AB = 62 yards, AC = 83 yards, angle A = 76° 28′. Are there two possible solutions?

36. Two rocks A and B lie due East and West of one another and are seven miles apart. From a ship A bears S. 24° W. and B bears S. 35° E.

How far is each rock from the ship?

37. The flagstaff outside a certain coastguard station is 24 feet in height and stands at the summit of a perpendicular cliff. From a boat the angles of elevation of the top and bottom of the flagstaff are observed to be 30° 53′ and 26° 44′ respectively. Find the distance of the boat from the foot of the cliff.

38. Two circles of radii 3 feet 6 inches and 2 feet 3 inches have their centres 4 feet apart. What is the angle between their common tangents?

39. P and Q are two points on a field such that Q is 66 yards N. 38° 15′ E. from P. From Q a tree bears N. 74° 21′ W., and from P the same tree bears N. 24° 8′ W.

(i) Find the distance of Q from the line joining P to the tree.

(ii) Find the distance of the tree from Q.

40. In a triangle ABC, $B = 110°$, $C = 20°$, $a = 158$ ft.

Find the side b, and the area of the triangle.

41. O is the centre of a circle, and AB a diameter 10 inches long. C is a point on the circumference. The straight line AC is 7·66 inches long. Find (i) the angle AOC, (ii) the length of the minor arc AC.

42. The position of an inaccessible point C is required.

From A and B, the ends of a base-line 220 yards long, the following bearings are taken :—

From A the bearing of B is N. 70° 30′ E. and the bearing of C is N. 30° 20′ E. From B the bearing of C is N. 59° 40′ W

Find the distances of C from A and B.

43. From a ship steaming N. 50° E. a lighthouse bears N. 30° E. Six minutes later the lighthouse bears N. 10° E. When will it be abeam and how near will the ship approach it? Speed of ship 10 knots.

44. In a simple steam engine the connecting rod CP is 15 feet 9 inches long and the crank 3 feet $4\frac{1}{2}$ inches. Find the angle at P when the distance CO is 17 feet.

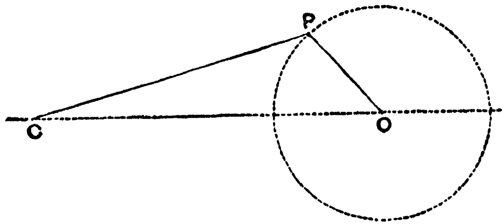


Fig. 24.

45. A rugby football ground is 76 yards wide, and the goal posts are 18 ft. 6 in. apart. Find the angle subtended by the line joining the goal posts at (i) the middle point of the 25-yard line, (ii) the point where the 25-yard line meets the touch line.

46. Two points A, B are taken on a level plain such that the distance between them is 1 mile. A point C on the same level is observed from A and B, and angles CAB and CBA are measured.

If $CAB = 40°$ and $CBA = 100°$, calculate the angle ACB and the lengths of CA and CB in miles.

47. If with the same base-line (AB) as in the last question a point D is observed and $DAB = 80°$ and $DBA = 70°$, calculate AD and DB.

48. What is the angle CAD in the figure of questions 46 and 47? By means of the triangle CAD, find the length of CD.

49. By means of the triangle CBD, find the length of CD.

(The answer should be the same as that of the last question.)

50. Using the results of the questions 46 to 49 above, show how CD can be found *in the shortest way* when $AB = 1$ mile and the angles at A and B are the same as before. Arrange the work very neatly and verify by drawing to scale.

51. A base line AB is 2 miles long; C and D are two points on the same level as AB and the angles at A and B are observed. Find CD if $ABC = 76°$, $ABD = 54°$, $BAC = 43°$, $BAD = 113°$. Arrange the work as in the last exercise.

52. In the triangle CAD of the last question calculate the angle ADC.

Now if A is known to be due West of B, find the bearing of C from D.

The process above is called *Triangulation* and is used in making maps of the country.

53. P is 800 yards due North of Q. If X is N. 73° 25′ E. of P and N. 39° 41′ E. of Q; and Y is S. 79° 20′ E. of P and N. 77° 12′ E. of Q, find the distance and bearing of X from Y. Verify by drawing to scale.

54. The angle of elevation of a tower 100 feet high due North of an observer was 50°. What will be its elevation when the observer has walked due East 300 feet?

55. The elevation of a balloon was observed at a certain station to be 20° and its bearing was N.E. At a second station 4000 yards due South of the former one its bearing was N. 11¼° E. Find its height.

56. The elevation of the top of a spire at one station A was 23° 50′ and the horizontal angle at the station between the spire and another station B was 93° 4′. The horizontal angle at B was 54° 28′ and the distance between the stations 416 feet. What was the height of the spire?

57. A ship was 2640 yards due South of a lighthouse. After the ship had sailed 800 yards N. 33¾° W. the angle of elevation of the top of the lighthouse was 5° 25′. Find its height.

58. If the jib of an ordinary crane is 15 feet long, the post is 10 feet high and the tie is 7 feet long, what angles do the jib and tie make with the post?

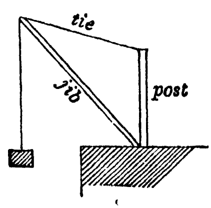


Fig. 25.

59. What must be the lengths of the tie in the crane above when the jib makes angles of 20°, 40°, 55° with the post?

60. What are the lengths of the diagonals of a parallelogram having adjacent sides 3 in. and 5 in. long and the included angle 40°?

61. If a parallelogram has two adjacent sides 10 cm. and 12 cm. long and the included angle 73°, what is the length of the diagonal passing through the intersection of the given sides?

62. If the included angle were 107° (instead of 73°), what would the length of the diagonal be?

63. From one corner O of a cube distances OA, OB, OC are measured along the edges so that OA = 2 in., OB = 3 in. and OC = 4 in. Find the angles of the triangle ABC.

64. From one corner A of a regular tetrahedron (see Fig. 9) distances AX = 5 cm., AY = 4 cm., AZ = 3 cm. are measured along the edges AB, AC, AD. Find the angles of the triangle XYZ.

65. From a point A due West of a hill TN the angle of elevation of the top T is found to be 43° 26′. From B which is 300 feet W. of A its elevation is 27° 39′. Find its height.

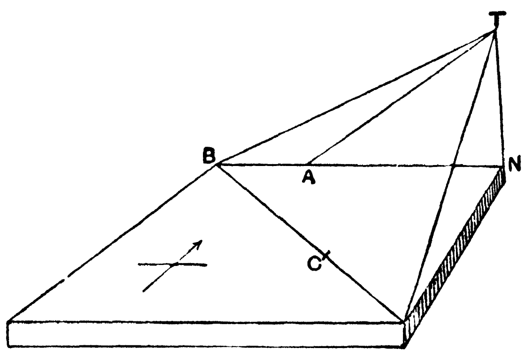


Fig. 26.

66. If C is 500 feet S.E. of B in question 65, what is the angle of elevation of T from C?

67. A path AB straight down the steepest slope of a smooth hillside makes an angle of 40° with the horizontal. If another straight path AC starting from the same point goes down the slope and makes an angle of 30° with the first path AB, what is its angular slope ∠ ACD?

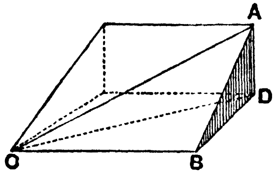


Fig. 27.

Find the ratios $\frac{AB}{AD}$, $\frac{AC}{AB}$ and hence $\frac{AC}{AD}$ and ∠ ACD.

68. If AB in the last question runs due South, in what direction (give compass bearing) does AC run?

Note that angles CAB, CDB are not equal. CDB gives the bearing.

69. A hillside slopes towards the South at a gradient of 1 in 2. In what direction (compass bearing) must a path be made up the hillside so that its gradient may be 1 in 5?

70. If a hillside slopes at an angle of 42° towards the North, what is the slope of a road running up it in a direction S. 73° E.?

71. A weight R is suspended by three strings OR, OP, OQ knotted together at O. If a parallelogram OBCA be drawn (as in the figure) with CB, CA parallel to PO, QO, and the diagonal OC in the same straight line as OR, then the tensions on the strings OP, OQ, OR are proportional to the lengths of OA, OB, OC.

If the weight R is 10 lbs. and the strings OP, OQ make angles of 30° and 70° with OC, show that the tensions on these strings are approximately 5·08 and 9·54 lbs. weight.

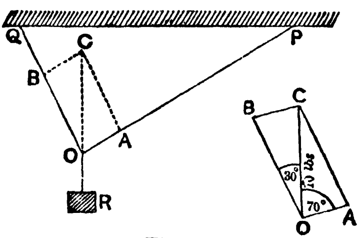


Fig. 28.

72. If R = 10 lbs. and each of the strings OP, OQ makes an angle of 35° with OC, what must the tensions on OP and OQ be?

73. If R = 12 lbs. and the tension on OP is 8 lbs. weight when ∠ POQ = 120°, what must be the tension on OQ, and what is the angle QOR?

74. What must the weight R be if the tension on OQ is 3 lbs. weight, the tension on OP is 5 lbs. weight, and the angle POQ is 130°?

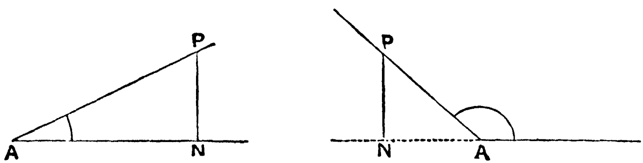
75. A station A is 1000 yards due West of another station B and on the same level. A point C is N. 20° E. of B and N. 50° E. of A and its elevation from B is 30°. Find the height of C above the level of AB and the angle subtended by AB at C.

APPENDIX A

DEFINITIONS AND RELATIONS BETWEEN RATIOS

I. If from any point P in one arm of an angle A a straight line PN be drawn perpendicular to the other arm, then

the ratio	$\frac{PN}{AP}$	is called the	*sine*	of the angle A,
"	$\frac{AN}{AP}$	" "	*cosine*	" "
"	$\frac{PN}{AN}$	" "	*tangent*	" "
"	$\frac{AP}{AN}$	" "	*secant*	" "
"	$\frac{AP}{PN}$	" "	*cosecant*	" "
"	$\frac{AN}{PN}$	" "	*cotangent*	" "



II. From the definitions above:

(1) $\operatorname{cosec} A = \frac{1}{\sin A}$, $\sec A = \frac{1}{\cos A}$, $\cot A = \frac{1}{\tan A}$.

(2) $\frac{\sin A}{\cos A} = \frac{\frac{PN}{AP}}{\frac{AN}{AP}} = \frac{PN}{AN} = \tan A$. Also $\frac{\cos A}{\sin A} = \cot A$.

(3) $\sin^2 A + \cos^2 A = \left(\frac{PN}{AP}\right)^2 + \left(\frac{AN}{AP}\right)^2 = \frac{PN^2 + AN^2}{AP^2} = \frac{AP^2}{AP^2}$

by Pythagoras' Theorem.

$$\therefore \sin^2 A + \cos^2 A = 1.$$

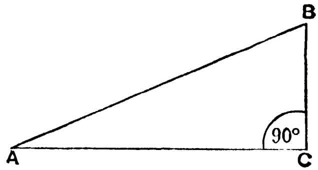
III. If ABC is a triangle with a right angle at C, the angles at A and B are *complementary* and $B = 90° - A$.

$$\sin B = \frac{AC}{AB} = \cos A,$$

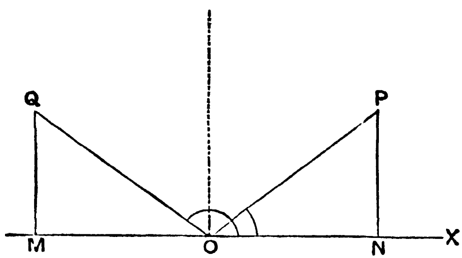
$$\tan B = \frac{AC}{BC} = \cot A,$$

$$\sec B = \frac{AB}{BC} = \operatorname{cosec} A.$$

Each ratio of an angle = the co-ratio of its complement.



IV. If an angle POX be continually increased by keeping OX in a fixed position and revolving OP, PN increases and ON decreases until POX becomes 90°, and then PN = OP and ON vanishes.



If OP continue to revolve so that POX becomes an obtuse angle, PN will begin to decrease and it is measured upwards from OX as before, but ON will begin to increase and is now measured in the *opposite* direction *away* from the angle.

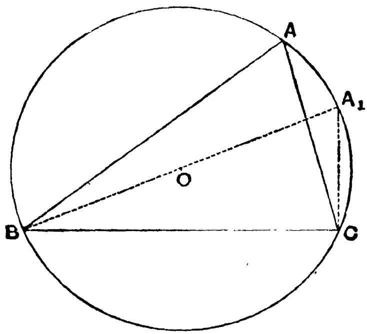
Thus the sine of an obtuse angle (QOX) is the same as the sine of its supplement (POX), while *the cosine of an obtuse angle is always negative* but its measure is the same as the cosine of its supplement.

APPENDIX B

SINE FORMULA

Let ABC be any triangle and O the centre of the circumscribing circle.

Draw a diameter BOA_1 and join A_1C.



The angles at A and A_1 are equal (in the same segment).

If R is the radius of the circle, then $BA_1 = 2R$.

Also $BCA_1 = 90°$ (in a semicircle).

$$\therefore \quad \frac{BC}{BA} = \sin BA_1C \quad \text{or} \quad \frac{a}{2R} = \sin A,$$

that is
$$2R = \frac{a}{\sin A}.$$

By joining AA_1 it may be shown in the same manner that

$$2R = \frac{c}{\sin C}.$$

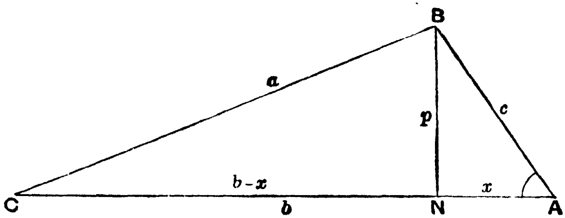
By drawing a diameter through A (or C) it may also be shown that

$$2R = \frac{b}{\sin B}.$$

$$\therefore \quad \text{in any triangle } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R.$$

APPENDIX C

COSINE FORMULA

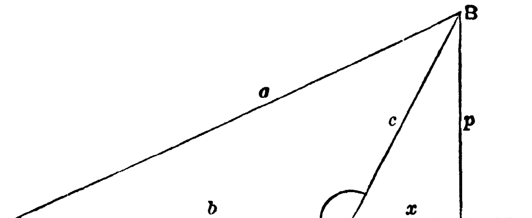


Let ABC be any triangle with an acute angle A.

Draw BN perpendicular to AC. Let its length be p units.

Suppose AN is x units long. Then AC is $b - x$ units.

Then $$\begin{aligned} a^2 &= p^2 + (b-x)^2 \quad \text{(by Pythag.)} \\ &= p^2 + b^2 + x^2 - 2bx \\ &= b^2 + c^2 - 2bx \quad (\text{for } c^2 = p^2 + x^2) \\ &= b^2 + c^2 - 2bc \cos \mathrm{A} \quad (\text{for } x = c \cos \mathrm{A}). \end{aligned}$$



If the angle A be obtuse, with the same construction as before CN will be $b + x$ units long.

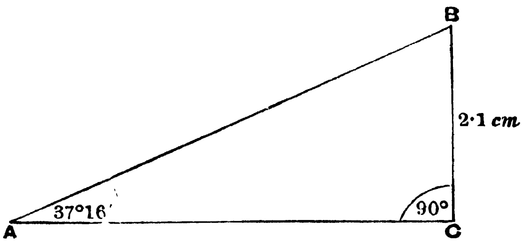
Then $$\begin{aligned} a^2 &= p^2 + (b+x)^2 \quad \text{(by Pythag.)} \\ &= p^2 + b^2 + x^2 + 2bx \\ &= b^2 + c^2 + 2bx \quad (\text{for } c^2 = p^2 + x^2) \\ &= b^2 + c^2 + 2bc \cos \mathrm{BAN} \\ &= b^2 + c^2 - 2bc \cos \mathrm{A} \quad (\text{for } \cos \mathrm{BAN} = -\cos \mathrm{A}). \end{aligned}$$

APPENDIX D

EXAMPLES OF ORDERLY ARRANGEMENT OF WORK

I. Solve the triangle ABC in which

$$\left.\begin{aligned} \text{C} &= 90^\circ \\ \text{A} &= 37^\circ\ 16' \\ a &= 2{\cdot}1 \text{ cm.} \end{aligned}\right\}.$$



$$\text{B} = 90^\circ - 37^\circ\ 16' = \underline{52^\circ\ 44'}.$$

$$\begin{array}{r} 13143 \\ 21 \\ \hline 26286 \\ 13143 \\ \hline 276003 \end{array}$$

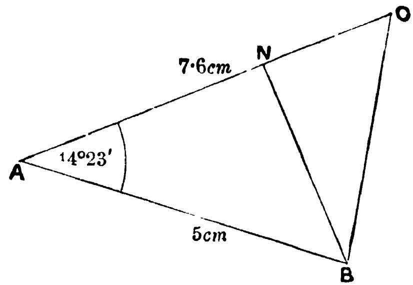
$$\begin{aligned} b = a \tan \text{B} &= 2{\cdot}1 \times 1{\cdot}3143 \\ &= \underline{2{\cdot}760 \text{ cm.}} \end{aligned}$$

$$\begin{array}{r} 16515 \\ 21 \\ \hline 33030 \\ 16515 \\ \hline 346815 \end{array}$$

$$\begin{aligned} c = a \operatorname{cosec} \text{A} &= 2{\cdot}1 \times 1{\cdot}6515 \\ &= \underline{3{\cdot}468 \text{ cm.}} \end{aligned}$$

II. Solution by dividing into right-angled triangles.

Given $\left.\begin{array}{l} A = 44^\circ\ 23' \\ b = 7{\cdot}6 \text{ cm.} \\ c = 5 \text{ cm.} \end{array}\right\}.$



Draw BN perpendicular to AC.

$$\begin{aligned} AN &= 5 \cos 44^\circ\ 23' \\ &= 5 \times 0{\cdot}7147 = 3{\cdot}5735. \\ \therefore \quad CN &= 7{\cdot}6 - 3{\cdot}5735 = 4{\cdot}0265 \text{ cm.} \end{aligned}$$

Also $$\begin{aligned} BN &= 5 \sin 44^\circ\ 23' \\ &= 5 \times 0{\cdot}6994 = 3{\cdot}497. \end{aligned}$$

But $$\tan C = \frac{BN}{CN} = \frac{3{\cdot}497}{4{\cdot}0265} = 0{\cdot}8688.$$

3·497	0·5437
4·026	0·6048
	$\bar{1}{\cdot}9389$

$\therefore \quad C = 40^\circ\ 59'.$

But $\quad A = 44^\circ\ 23'.$

$\therefore \quad B = 94^\circ\ 38'.$

(Check $A + B + C = 180^\circ\ 0'$.)

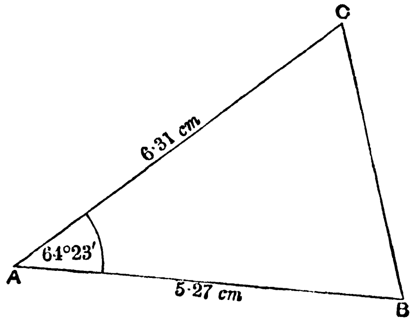
Also $$\begin{aligned} CB &= \sqrt{BN^2 + CN^2} \\ &= \sqrt{12{\cdot}23 + 16{\cdot}21} = \sqrt{28{\cdot}44} \\ &= 5{\cdot}33 \text{ cm.} \end{aligned}$$

$$\left\{\begin{array}{l} a = 5{\cdot}33 \text{ cm.} \\ B = 94^\circ\ 38. \\ C = 40^\circ\ 59'. \end{array}\right.$$

III. Solution by formulae.

Given

$$\left.\begin{aligned} A &= 64^\circ\ 23' \\ b &= 6{\cdot}31 \text{ cm.} \\ c &= 5{\cdot}27 \text{ cm.} \end{aligned}\right\}.$$



$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cos A \\ &= 6{\cdot}31^2 + 5{\cdot}27^2 - 2 \times 6{\cdot}31 \times 5{\cdot}27 \cos 64^\circ\ 23' \\ &= 39{\cdot}82 + 27{\cdot}77 - 28{\cdot}75 \\ &= 67{\cdot}59 - 28{\cdot}75 \\ &= 38{\cdot}84. \end{aligned}$$

2	0·3010
6·31	0·8000
5·27	0·7218
cos 64° 23′	$\bar{1}$·6358
	1·4586

$$\therefore\quad a = 6{\cdot}232 \text{ cm.}$$

Also $$\sin C = c\,\frac{\sin A}{a} = \frac{5{\cdot}27 \times \sin 64^\circ\ 23'}{6{\cdot}232}.$$

5·27	0·7218
sin 64° 23′	$\bar{1}$·9551
	0·6769
6·232	0·7946
	$\bar{1}$·8823

$\therefore$ C = 49° 42′ or its supplement.

But C must be acute, for c is not the longest side.

$$\begin{aligned} \therefore\quad C &= 49^\circ\ 42'. \\ \text{But}\quad A &= 64^\circ\ 23', \\ \therefore\quad B &= 65^\circ\ 55'. \end{aligned}$$

(Check A + B + C = 180° 0′.)

$$\left\{\begin{aligned} a &= 6{\cdot}23 \text{ cm.} \\ B &= 65^\circ\ 55'. \\ C &= 49^\circ\ 42'. \end{aligned}\right.$$

ANSWERS

The lengths have in general been calculated to four significant figures and the angles to the nearest minute. Slight discrepancies will often appear if the methods of calculation differ from one another.

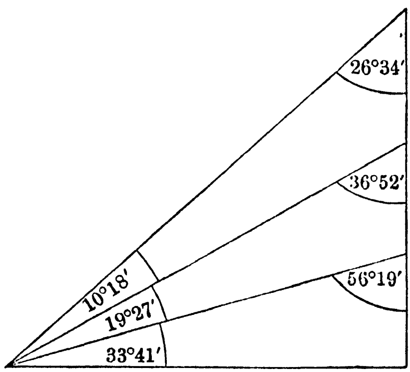
Chap. I

1. 180°, 45°, 30°, 6°, 18°. 2. 22° 30′, 7° 30′, 33° 45′, 64° 17′.
3. 53° 25′, 77° 0′, 58° 34′, 139° 46′.
4. 67° 50′, 58° 45′, 60° 35′, 42° 25′, 51° 9′, 70° 17′, 10° 3′, 30° 11′.
5. 126° 35′, 103° 0′, 121° 26′, 40° 14′. 6. 60°, 23°, 46° 39′.
7. 36°, 72°, 72°; 20°, 80°, 80°; 45°, 45°, 90°.
8. 72°, 108°, 72°.
9. (1) 45°, 135°, 45°. (2) 40°, 140°, 140°. (3) $51\frac{3}{7}$°.
10. S. 31° W. 11. 90°. 12. N. 13° E.
13. S. 45° E. 14. 24°. 15. 37°.
16. 5°. 17. 60°, 30°. 18. $\frac{2}{3}$, $\frac{1}{3}$, $\frac{1}{6}$.
19. ABP = 48°. 20. $x° - y°$.
21. 60°, 60°, 90°, 30°. CK = 5 cm. AK = 8·66 cm AKN is greater than ACN. AKN decreases.
22. It varies, and is greatest when equidistant from the two points.
23. The circumference of the circle passing through the ship and the two points.

Chap. II

13. 1·4826, 1·5051, 1·5108, 1·5080;
0·5317, 0·5362, 0·5384, 0·5373;
0·2946, 0·3476, 0·0673.
15. 25°, 72°, 17° 30′, 55° 12′, 21° 55′, 74° 50′, 11° 10′, 66° 39′, 50° 9′, 33° 33′.
16. tan 35° = 0·7002. 17. 38° 40′, 63° 26′, 74° 3′.

18. 55·43 feet.
19. 47·67 feet.
20. 67° 23′, 22° 37′, 13 feet.
21. S. 35° 32′ E., 8·602 miles.
22. 134·9 feet.
23. 30° 58′.
24. 5·095.
25. 5·139.
26. 5·154.
27. 1·288.
28. 2·690.
29. 127·5.
30. 38° 40′.
31. A = 38° 40′, B = 51° 20′.
32. A = 58°, B = 32°.
33. A = 34° 21′, B = 55° 39′.
34. B = 47° 43′, $a = 23·10$, $c = 34·34$.
35. A = 60° 49′, $b = 0·9048$, $c = 1·856$.
36. B = 76° 7′, $b = 42·08$, $c = 43·34$.
37. A = 17° 19′, $a = 1·269$, $c = 4·262$.
38. A = 53° 8′, B = 36° 52′, $c = 15·5$.
39. A = 39° 42′, B = 50° 18′, $c = 194·9$.
40. 54 feet.
41. 248·6 feet.
42. 1003 feet.
43. 69·88 yards.
44. 167·3 feet.
45. 28·8 in.
46. 112° 37′.
47. 1·82 cm.
48. 88·18, 59·62 feet.
49. 451·3 yards, 12° 48′.
50.



51. 42·88 secs.
52. 193·0 feet.
53. 396·2 feet.
54. 93·26 feet.
55. 57·82 feet.
56. 45°, 54° 44′.
57. 8·165 cm., 54° 44′, 70° 32′.

Chap. III

11. 0·5764, 0·5779, 0·5771, 0·5766; 0·9559, 0·6839, 0·1888, 0·9898.
13. 25°, 71°, 54° 30′, 16° 48′, 16° 51′, 78° 23′, 75° 57′, 46° 4′.
14. 30°. 15. 23° 35′, 48° 36′, 25° 28′.
16. 4·54, 4·571, 4·581, 1·145, 8·416, 2·210.
17. $b = 67{\cdot}10$, $a = 33{\cdot}51$, $A = 26° 32'$.
18. (1) $B = 17° 46'$, $a = 20$, $b = 6{\cdot}407$.
(2) $A = 64° 37'$, $a = 9{\cdot}395$, $b = 4{\cdot}457$.
(3) $A = 47° 47'$, $a = 77{\cdot}91$, $b = 70{\cdot}68$.
(4) $B = 78° 41'$, $a = 3{\cdot}791$, $b = 18{\cdot}95$.
19. 2° 18′. 20. 0° 34′. 21. 0° 46′, 1° 26′.
22. 23 ft. 11 in. 23. 0·755 in. 24. 4·23 mi.
25. 6·47 in. 26. 32° 14′. 27. 3 ft. 1 in.
28. (1) 0·771, 1·845. (2) 1·909, 0·595.

Chap. IV

2. 0·515, 0·777. 3. 60°, 41·4°, 68·9°.
6. 0·5592, 0·5534, 0·5519; 0·5526(7).
7. 0·8809, 0·9592, 0·2866, 0·1417, 0·5999, 0·9764.
8. (1) 8·91. (2) 7·27. (3) 8·30. (4) 1·003.
9. 44·65 feet. 10. 14·85 mi. 11. 1736 yards.
12. 12·6 feet. 13. 4·77 in. 14. 69·09 mi.
15. 1·054 feet. 16. 7·5 cm. 17. 9·34 feet.
18. 5 ft. 6·1 in. 19. 3064 mi., 19260 mi. 20. 10·86.
21. Half the length of the Equator. Half.

Chap. V

5. $\operatorname{cosec} A = \frac{c}{a}$, $\sec A = \frac{c}{b}$, $\cot A = \frac{b}{a}$.

6.

Angle	56° 27′	28° 14′	16° 25′	73° 21′
cosec	1·1999	2·1140	3·5383	1·0437
sec	1·8094	1·1351	1·0425	3·4903
cot	0·6631	1·8624	3·3941	0·2991

7. As angle increases, sec increases, cosec and cot decrease.
8. 22·65, 33·48, 42·55 ft. 9. 32·86 ft. 10. 1191 ft.
11. 10·21 cm. 12. 11·33 ft. 13. 47·91 in.
14. 156·7 ft. 15. 6·22 mi. 16. 49·05 in.
17. 18·70 ft. 18. 11·63 in. 19. 1 in 14.
20. 20·27 ft. 21. $A = 36° 52'$, $B = 53° 8'$.
22. $A = 77°$, $B = 13°$.
23. $\sin A = \cos B$, $\cot A = \tan B$, $\sec A = \operatorname{cosec} B$.
24. 22° 37′. 25. 16° 16′.
26, 27. $\dfrac{\sin A}{\cos A} = \tan A$, $\sin^2 A + \cos^2 A = 1$.

28.
$$\frac{\sin A}{\cos A} = \frac{\frac{a}{c}}{\frac{b}{c}} = \frac{a}{b} = \tan A,$$

and
$$\sin^2 A + \cos^2 A = \frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{a^2 + b^2}{c^2} = \frac{c^2}{c^2} = 1.$$

30. $\sin 90° = 1$, $\cos 90° = 0$, $\tan 90° = \infty$ (the nearer 90°, the larger), $\sin 0° = 0$, $\cos 0° = 1$, $\tan 0° = 0$.
32. sine and cosine must be less than 1. Tangent may have any value.
33—5. $\sec^2 A - \tan^2 A = 1$. 36. $\operatorname{cosec}^2 A - \cot^2 A = 1$.

Chap. VI

1. $\sin B = \dfrac{p}{a} = \dfrac{y}{b} = \dfrac{b}{c}$. 2. $\cos B = \dfrac{x}{a} = \dfrac{p}{b} = \dfrac{a}{c}$ etc.
3. $\sin A = \dfrac{x}{a} = \dfrac{p}{b} = \dfrac{a}{c}$ etc.
4. $B = 36° 52'$, $A = 53° 8'$, $b = 15$, $x = 16$, $y = 9$, $c = 25$ cm.
5. $A = 67° 23'$, $B = 22° 37'$, $a = 15{\cdot}6$, $x = 14{\cdot}4$, $y = 2{\cdot}5$, $c = 16{\cdot}9$ cm.
6. $p = x \sin \alpha$, $y = x \tan \alpha$, $AC = x \sec \alpha$; $CN = p \tan \alpha = x \sin \alpha \tan \alpha$.
7. $a \cos B = x$, $b \sin A = p$, $c \cos B = a$, $a \cos A = p$, $p \tan B = y$, $p \cot B = x$, $p \sec A = a$, $p \operatorname{cosec} A = b$, $b \sin B = y$.
8. $x \sin \alpha = p$, $x \tan \alpha = y$, $p \cot \alpha = AN$, $y \cos \alpha = p$, $p \tan \alpha = CN$.

9. A $= 35° 41'$, $b = 4·700$, $c = 11·41$.
10. B $= 60° 7'$, $b = 34·98$, $c = 40·34$.
11. B $= 45° 29'$, $b = 1·711$, $a = 1·683$.
12. A $= 59° 25'$, B $= 30° 35'$, $c = 28·7$.
13. A $= 33° 22'$, B $= 56° 38'$, $c = 116·9$.
14. B $= 17° 47'$, $a = 48·33$, $c = 50·75$.
15. A $= 55° 23'$, $a = 987·5$, $b = 681·6$.
16. B $= 30°$, A $= 60°$, $a = 9·873$.
17. A $= 11° 32'$, B $= 78° 28'$, $b = 16·42$.
18. A $= 19° 28'$, B $= 70° 32'$, $b = a \times 2·828$.
19. 821·5 ft. 20. S. 55° 47′ W. 21. 67·02 ft.
22. 177·5 ft. 23. 0° 57′. 24. 8·274 cm.
25. 2·157 cm. 26. 30° 14′. 27. 7·779 cm.
28. 9·398 cm. 29. 8·402 cm., 42° 48′. 30. 181·4 yds.
31. 6·824 in. 32. 53° 8′. 33. 83·25 in.
34. 3·441 cm. 35. 4·253 cm. 36. 4·330, 5 cm.
37. 7·694, 8·090 cm. 38. 8·83 mi. per hr.
39. 54 ft. 0 in., 60 ft. 4·5 in. 40. 69° 41′.
41. 36° 52′, 66° 25′. 42. 50° 12′. 43. 27° 54′.
44. 34° 58′. 45. 8·201 mi. S. 52° 26′ E.
46. 52° 0′, 68° 40′, 75° 24′. 47. 15·64 cm.
48. 16° 42′, 17° 28′. 49. 20 ft. 1 in. 50. 747 ft.
51. 4650 fath. 52. 90°, 66° 25′, 9·165 in.
53. AO $= 11·92$, BO $= 8·39$, AC $= 15·56$, BC $= 13·05$ inches.
54. 1·595. 55. 18·03 mi. S. 71° 43′ W.
56. 5·362, 7·332 mi. 57. 83·39 ft. 58. 1132 ft.
59. 0° 27′. 60. 4·189 mi. 61. 51·5 ft., 354·6 ft.
62. 190 ft. 63. 76·9 yds. 64. S. 70° E.
65. 140, 141 ft. 66. 11·18 mi. S. 23° 37′ W.
67. 5·342 in., 0·395 in.
68. 4·35 mi. N. 38° 28′ E. or S. 38° 28′ E. 69. 30° 21′.
70. 297·9, 268·6, 29·2 ft. 71. 5 mi. N. 53° 8′ W.
72. 9·063, 4·226 mi. 73. 2·588 mi.
74. 16·73 knots, 18·99 mi.
75. 2·908 in., C $= 68° 12'$, A $= 74° 56'$. 76. 65·8 ft.

77. 63·74 ft. 78. 0·515, 0·4415 mi. 79. 111° 53′.
80. 30° 58′. 81. 3·215 mi. 82. 54·63, 64·97 ft.
83. 1125 ft. 84. 1·884, 5·73, 6·85 mi.
85. 65° 23′, 53° 54′, 62° 43′. 86. 3·535 in., 45°, 54° 44′.
87. 59° 0′, 25° 22′, 16° 36′. 88. 686·9, 1069 ft., 13° 10′.
89. 11° 48′. 90. 382 yds.

Chap. VII

1. 50 sq. cm., 13297·68 sq. yds.
2. 25 sq. cm., 6648·84 sq. yds. 3. 7·5, 6, 13·5 sq. cm.
4. Half the product of the number of cm. in BC and AN gives number of sq. cm. in area.
5. 3·214, 4·50 cm., 11·25 sq. cm.
6. Product of number of cm. in base and number of cm. in perpendicular height gives number of sq. cm. in area.
7. Area $= xy \sin A$. 8. $\frac{1}{2}xy \sin A$.
9. 5 sq. cm., 9 sq. cm., 16·9 sq. cm., 118 sq. cm.
10. 28·7 sq. cm. 11. 259·8 sq. cm.
12. 237·8, 273·7, 282·8, 289·3 sq. cm
13. 363·2, 346·4, 327·4 sq. cm. 14. 36 yds.
15. 53° 8′, 126° 52′, 4·268 cm. 16. 78° 42′, 22° 36′.
17. Sq. pyramid, 273·2 sq. cm. Tetrahedron, 173·2 sq. cm.
18. Segment, 25·67 − 21·22 = 4·45 sq. cm. 19. 15·3 sq. cm.
20. 89·04 sq. cm.

Chap. VIII

2. $10^{\frac{1}{2}} = \sqrt{10} = 3{\cdot}162$. 3. $10^{\frac{1}{4}} = \sqrt[4]{10} = \sqrt{3{\cdot}162} = 1{\cdot}779$.
4. $10^{\frac{3}{4}} = 5{\cdot}623$.
7. $3{\cdot}7 = 10^{0{\cdot}5682}$, $3{\cdot}75 = 10^{0{\cdot}5740}$, $3{\cdot}758 = 10^{0{\cdot}5749}$, $7{\cdot}64 = 10^{0{\cdot}8831}$, $7{\cdot}649 = 10^{0{\cdot}8836}$, $9{\cdot}8 = 10^{0{\cdot}9912}$, $9{\cdot}805 = 10^{0{\cdot}9914}$.
8. 2, 7·6, 2·75, 2·76, 2·757, 6·16, 6·17, 6·167, 4·486, 7·0.
9. 4·318. 11. 8·572, 8·179, 2·999, 6·451, 8·164, 9·936.
12. 2·940. 13. 2·397, 1·263, 3·855, 1·408, 2·752, 3·766.
16. Logs are 2·6660, 1·8986, 1·2119, 2·8581, 1·8581, 3·4934, 4·8825, 5·3109, 1·8664, 2·3294.

17. 710·1, 33, 3050, 888, 278000, 7115, 337·2, 26·92.

18.	(1) 37·62.		(2) 1390.		(3) 1143.	
	(4) 149000.		(5) 9243.		(6) 87000.	
	(7) 804·2.		(8) 1023.		(9) 2637000.	
	(10) 317800000.		(11) 5·690.		(12) 1·375.	
	(13) 463·1.		(14) 266·1.		(15) 1·395.	
	(16) 155·2.		(17) 4083.		(18) 2·867.	
	(19) 1·006.		(20) 9·779.			

20. Logs are $\bar{2}\cdot0835$, $\bar{1}\cdot0835$, $\bar{1}\cdot5518$, $\bar{4}\cdot5518$.

22. Logs are $\bar{2}\cdot5378$, $\bar{1}\cdot8860$, $\bar{3}\cdot6656$, $\bar{4}\cdot0433$, $\bar{1}\cdot9987$, $\bar{1}\cdot9573$, $\bar{2}\cdot6385$, $\bar{1}\cdot7350$.

24.	(1) 0·003762.	(2) 0·01490.	(3) 0·01143.
	(4) 0·001390.	(5) 92·43.	(6) 1·023.
25.	(1) 0·005690.	(2) 46·31.	(3) 0·003758.
	(4) 137·5.	(5) 0·1031.	(6) 10·28.

26. 3·684. 27. 17·88, 4·236, 10·45, 49·91, 6·223.

28. $\sqrt{0\cdot002} = (10^{\bar{3}\cdot3010})^{\frac{1}{2}} = 10^{\bar{2}\cdot6505} = 0\cdot04472$.

29.	(1) 0·2.	(2) 0·06325.	(3) 0·02.
	(4) 0·2.	(5) 0·09283.	(6) 0·4309.
	(7) 0·8747.	(8) 0·3849.	(9) 0·7830.
	(10) 0·1968.		

Miscellaneous Exercises in Logarithms

1. 1888.	2. 35·62.	3. 6·811.
4. 0·7492.	5. 305·8.	6. 31620.
7. 5·473.	8. 2·290.	9. 0·3098.
10. 0·0003321.	11. 0·08680.	12. 0·3607.
13. 0·0412.	14. 0·03728.	15. 6·725.
16. 6092.	17. 0·1460.	18. 0·6015.
19. 1531.	20. 64·92.	21. 9·177.
22. 165·8.	23. 13·36.	24. 18·84.
25. 40·15.	26. 1·501.	27. 98·88.
28. 0·5399.	29. 96·22.	30. 0·4846.
31. 0·461.	32. 16·13.	33. 10·93.
34. 110·4.	35. 0·4066.	36. 1·749.

Chap. IX

A. 1. 9·539 cm., 65° 12′, 54° 48′.

4. 7·5 cm., 53° 8′, 73° 44′.

5. 4·903 cm., B = 54° 40′, C = 101° 45′.

6. Drop perpendicular on to either of the given sides.

7. (1) $c = 17{\cdot}35$ cm., A = 72° 24′, B = 66° 11′.
   (2) $b = 2{\cdot}51$, A = 65° 11′, C = 53° 25′.
   (3) $a = 19{\cdot}39$, B = 71° 30′, C = 84° 13′.

8. $a = 8{\cdot}888$, B = 103°, C = 17°.

10. $a = 15{\cdot}69$, B = 30° 39′, C = 22° 29′.

12. (1) $a = 15{\cdot}92$, B = 115° 39′, C = 26° 33′.
    (2) $c = 129{\cdot}7$, A = 102° 4′, B = 19° 33′.
    (3) $c = 53{\cdot}78$, A = 20° 13′, B = 24° 31′.
    (4) $a = 5{\cdot}977$, C = 46° 20′, B = 35° 17′.
    (5) $b = {\cdot}6333$, C = 12° 50′, A = 5° 23′.

B. 1. C = 67° 23′, $a = 8{\cdot}666$, $b = 9{\cdot}333$.

2. AN = 8, NB = 6, NC = 3·333.

3. If AL be the perp., find angle C, AL, BL, CL, CA, CB.

4. Draw perp. from either end of the given side.

5. From B or C. If BN perp., find C, CN, NB, NA, AB, AC.

6. (1) C = 75° 24′, $c = 4{\cdot}925$, $b = 4{\cdot}458$.
   (2) C = 42° 59′, $a = 23{\cdot}54$, $b = 22{\cdot}52$.
   (3) C = 41° 50′, $a = 110\ 6$, $c = 150{\cdot}9$.
   (4) B = 125° 10′, $b = 50{\cdot}69$, $c = 40{\cdot}83$.
   (5) B = 19° 54′, $a = 63{\cdot}71$, $c = 79{\cdot}86$.

C. 1. $x = 0{\cdot}5$ cm., A = 55° 47′, B = 82° 49′, C = 41° 24′.

4. (1) A = 44° 3′, B = 52° 37′, C = 83° 20′.
   (2) A = 104° 29′, B = 46° 34′, C = 28° 57′.
   (3) A = 51° 19′, B = 110° 29′, C = 18° 12′.
   (4) A = 38° 13′, B = 60°, C = 81° 47′.
   (5) Perp. from B, AN = 1·827. A = 58° 32′, B = 93° 23′, C = 28° 8′.

D. 1. B $= 48° 36'$ or $131° 24'$, C $= 94° 32'$ or $11° 44'$, $c = 13{\cdot}29$ or $2{\cdot}708$.

3. Only from B. 4. B $= 30°$, C $= 113° 8'$, $c = 18{\cdot}39$.

5. When the side opposite the given angle is less than the other given side.

E. 1. B $= 52° 22'$, C $= 73° 15'$, $a = 19{\cdot}96$.

2. C $= 27° 15'$, $a = 9{\cdot}058$, $b = 5{\cdot}907$.

3. B $= 59° 24'$, $120° 36'$, C $= 78° 55'$, $17° 45'$, $c = 30{\cdot}44$, $9{\cdot}439$.

4. A $= 33° 19'$, B $= 92° 38'$, C $= 54° 3'$.

5. A $= 45° 52'$, C $= 70° 42'$, $c = 120{\cdot}3$.

6. A $= 75° 59'$, $b = 1{\cdot}462$, $c = 3{\cdot}521$.

7. A $= 70° 36'$, B $= 62° 5'$, $c = 148{\cdot}8$.

8. A $= 51° 45'$, B $= 54° 32'$, C $= 73° 43'$.

9. A $= 55° 50'$, $124° 10'$, B $= 86° 47'$, $18° 27'$, $b = 149{\cdot}6$, $47{\cdot}72$.

10. B $= 56° 12'$, $a = 7{\cdot}104$, $c = 8{\cdot}529$.

11. A $= 131° 21'$, B $= 28° 0'$, C $= 20° 39'$.

12. C $= 48° 44'$, $b = 15{\cdot}59$, $c = 11{\cdot}91$.

Chap. X

1.

Angle	36° 52′	53° 8′	60°	120°	126° 52′	143° 8′
ON	8	(5·9999 say) 6	5	5	6	8 cm.
AN	25	23	22	12	11	9 cm.

The cosine of each of these obtuse angles is negative.

2. Supplementary angles. $-0{\cdot}5$, $-0{\cdot}6$, $-0{\cdot}8$. Cosine of an obtuse angle $= -$(cosine of its supplement).

3. PN $= 6$, 8, $8{\cdot}66$, $8{\cdot}66$, 8, 6 cm. Supplementary angles. $0{\cdot}8660$, $0{\cdot}8000$, $0{\cdot}5999$.

4. Sine $= \frac{12}{13}$, tangent $= -\frac{12}{5}$, secant $= -\frac{5}{13}$, cosecant $= \frac{13}{12}$, cotangent $= -\frac{5}{12}$.

5. sin $0{\cdot}6428$, $0{\cdot}8387$, $0{\cdot}9694$, $0{\cdot}2275$, $0{\cdot}4638$.
cos $-0{\cdot}7660$, $-0{\cdot}5446$, $-0{\cdot}2456$, $-0{\cdot}9738$, $-0{\cdot}8859$.

6. tan $-0{\cdot}8391$, $-1{\cdot}5399$, $-3{\cdot}9474$, $-0{\cdot}2336$, $-0{\cdot}5235$.

7. cot −1·1918, −0 6494, −0·2533, −4·2812, −1·9101
sec −1·3054, −1·8361, −4·0720, −1·0269, −1·1287.
cosec 1·5557, 1·1924, 1·0316, 4·3965, 2·1560.

8. 0·8988, 0·2756, 0·5948, −0·8949,
0·9802, −0·5233, 0·9324, −0·9951,
0·8811, −0·7988, 2·8478, −0·1646,
1·9040, 1·0234, −1·5037, 1·0807.

9. 50° and 130°.

10. 64° 10′ or 115° 50′, 128° 30′, 16° 37′ or 163° 23′,
82° 58′, 97° 2′, 19° 28′ or 160° 32′,
69°, 111°, 103° 30′,
61° 28′ or 118° 32′, 71° 14′, 108° 46′.

11. The sine is positive. The cosine is positive for acute, negative for obtuse angles.

Chap. XI

1. C = 60°, b = 15·32 cm., c = 13·47 cm.

4. The ratio of two sides = the ratio of sines of opposite angles.

6. 8·830. 7. 6·979, 8·412, 3·034, 2·966, 5·964.

8. $\frac{\sin A}{\sin B} = \frac{3}{10}$. 9. $CN = b \sin A = a \sin B$.

11. (1) 8·333. (2) 8·50. (3) 4·069.
(4) 14·77. (5) 42·61. (6) 74° 49′, 7·139.
(7) 1·515. (8) 3·780.
(9) 36° 52′ or 143° 8′. (10) 48° 28′ or 131° 32′.
(11) 49° 23′ or 130° 37′. (12) 30° 24′.

12. 30°, 8·134 cm., 6·580 cm. 13. $200 \cdot \frac{\sin 61° \sin 41°}{\sin 20°} = 335·5$.

14. 93·26 feet. 15. 193 feet. 16. 57·82 feet.

Chap. XII

1. 1·565, 2·572, 3·606, 4·564.

5. $a^2 = b^2 + c^2 - 2bc \cos A$, $b^2 = c^2 + a^2 - 2ca \cos B$,
$c^2 = a^2 + b^2 - 2ab \cos C$.

7. 12·81, 14·78, 16·33, 17·40, 17·94.

9. (1) 2·661. (2) 8·655. (3) 9·495. (4) 11·59.
(5) 3·465. (6) 13·39. (7) 38·92.

10. (1) A = 41° 25′, B = 55° 46′, C = 82° 49′.
(2) 44° 25′, 57° 7′, 78° 28′.
(3) 53° 8′, 90°, 36° 52′. (4) 119° 5′, 30° 51′, 30° 4′.

11. 29° 55′. 12. 108° 13′. 14. 7·76 miles. 15. 83° 6′.

Chap. XIII

1. 1588, 551, 491 yds. 2. 2019, 3445 yds.
3. 1985 yds. 4. 4·82 mi. 5. 1119, 817, 717 ft.
6. 11·0 ft. 7. 98·1 ft.
8. (1) 5° 51′, 88·0 in. (2) 14° 29′, 69·7 in.
(3) 11° 58′, 60·4 in.
9. (1) 64° 10′, 78·0 in.; 115° 50′, 62·3 in.
(2) 46° 7′, 83·3 in.; 133° 53′, 58·3 in.
10. 49·75 ft. 11. 12·97, 11·33 mi. 12. 403 yds.
13. 4·54 cm. 14. 9·164, 63° 55′. 15. 28° 21′.
16. 3·23 ft., 19° 48′. 17. 4·877 mi., N. 57° 35′ W., 3·982 mi.
18. 50·10, 100·6 ft. 19. 270·9 yds.
20. A = 127° 29′, C = 20° 31′, b = 18·9 cm.
21. c = 123 yds. Area = 4863 sq. yds. 22. 95° 44′.
23. 393·6 yds., S. 32° 10′ E., 739·4 yds.
24. 5·29 mi., N. 74° 26′ E. or S. 74° 26′ W.
25. 3·351 in., 37° 18′, 75° 42′. 26. 10·13 cm.
27. 58·48 sq. in., 7·463, 18·98 in. 28. 3·834, 4·819 mi.
29. 27·67, 35·47 ft. 30. 15·98 mi. 31. 667 ft.
32. A = 35° 41′, B = C = 72° 9′, AB = AC = 5·711 cm.
33. 3·7 cm. 34. 763·2, 1203 yds.
35. (1) C = 36° 47′ or 143° 13′, B = 115° 13′ or 8° 47′, b = 35·66 or 6·019 in.
(2) a = 91·24 yds., B = 62° 11′, C = 41° 21′.
36. 6·689, 7·459 mi. 37. 254·5 ft. 38. 36° 25′.
39. 58·48, 76·08 ft. 40. 194 ft., 5240 sq. ft.
41. 100°, 8·7 in. 42. 168, 142 yds.

43. 4 min. 36 secs., 0·642 miles. **44.** 106° 10′.
45. 14° 4′, 4° 17′. **46.** 1·532 mi., 1 mi.
47. 1·879, 1·970 mi. **48.** 1·210 mi. **49.** 1·212 mi.
51. 6·76 mi. **52.** S. 41° E. to within 5′.
53. PX = 919·8, PY = 1959, QX = 1380, QY = 1973. XY from triangle PXY = 1217, and from triangle QXY = 1214 yds.
54. 19° 9′. **55.** 511·2 yds. **56.** 278·6 ft.
57. 192 yds. **58.** 23° 4′, 57° 7′.
59. 6·564, 9·757, 12·37 ft. **60.** 3·32, 7·55 in.
61. 17·73 in. **62.** 13·18 in.
63. 75° 38′, 60° 3′, 44° 19′. **64.** 69° 32′, 63°, 47° 28′.
65. 351·9 ft. **66.** 36° 28′. **67.** 33° 50′. **68.** S. 37° W.
69. N. 69° 18′ E. or W. **70.** 14° 45′ or about 1 in 4.
72. 6·104 lbs. wt. **73** 13·8 lbs. wt., 144° 44′.
74. 3·84 lbs. wt. **75.** 742·3 yds., 28° 16′.

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